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PREFACE.

THIS book is the outgrowth of the difficulties experienced by the author in teaching the first elements of instrumental practice to engineering students. The frequent practice of taking a dozen or twenty students into the field to do a piece of work which can hardly keep more than four or five men busy results generally in the work being really done by the few energetic ones who are anxious to learn, while the rest stand around and pick up what they can—which is frequently an insignificant quantity.

These problems, which have had the benefit of revision after two years' use, are designed to keep each student busy at some definite work, for which he has definite instructions, and to utilize so thoroughly the limited time usually available for this work, that he will have a working knowledge of all the more common engineering instruments by the time he is ready for more advanced work. The student's attention is called to the degree of precision that is required in instrumental work in order to obtain any required precision in the final results—a feature often neglected. Systematic note-keeping, for which forms are given, is required of the students.

This book is not intended to replace any general text-book on Surveying. It aims to supplement the general instructions of such text-books by such definite directions that the students may work *alone*, aided only by such occasional help as may be given by one instructor looking after several parties, and thus solving the vexing problem of properly looking after the work of a large number of students with a limited corps of instruction. The work of many of the problems (e.g., chaining, levelling, angle measurements, etc.) may be so systematized by means of permanent hubs on the surveying ground that the instructor may instantly judge of the character of the work done by a mere inspection of the notes submitted.

As these instructions were written primarily for the use of the *students of* the University of Pennsylvania, they contain some loc allusions, but even where these occur it will be very simple to use the directions in a general way and apply them to any locality.

Of course the number of these problems could be increased indefinitely, but as the time available for this work is generally somewhat limited, it is deemed best to give the student only such problems as will most speedily and effectually teach him the capabilities and limitations of his instruments. Any student who works out satisfactorily the full list of these problems will have a better working knowledge of engineering instruments than many graduates just out of college.

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GENERAL INSTRUCTIONS.

1. Complete notes of each problem, in proper form, tabulated whenever possible, must be made during the progress of the field work.

2. The notes should be so complete that the significance of every recorded number should be easily intelligible to any one conversant with the subject.

3. All calculations must be completed when reports are submitted.

4. Note-books must be turned in each day with the instruments—except when the problem requires extended computations on the data obtained in the field, in which case two days' extra time will be allowed.

5. The note-book should be a journal of each day's field-work—giving (*a*) date, (*b*) time of beginning and ending work, (*c*) number of problem, (*d*) *personnel* of party, and then a detailed report of individual work.

6. No notes are to be copied from another's note-book, except such figures as target readings of a level rod, etc., taken by the assistant, which will be necessary for a complete report on the problem. With such exceptions each student reports *only* his own individual work.

7. When a problem requires the work of two or more (as is usually the case) *each* student in turn must act as chief, do the instrumental work, and take notes for a *complete* solution of the problem.

8. Even when a student acts only as assistant during any one day's work, a report must be submitted by him giving date, time of work, number of problem, and *personnel* of party.

9. No credit will be given for field-work except as it can be estimated from the notes. No verbal reports will be credited.

10. Read the instructions carefully *before* beginning the field-work on any problem; be sure that all necessary equipment is provided and that the instructions are understood.

11. Students are held responsible for all equipment issued to them. Therefore, examine all equipment as it is issued and report *immediately any injury* or deficiency that may be discovered, in order that *the responsibility* may be properly located.

PROBLEMS IN THE USE AND ADJUSTMENT

OF

ENGINEERING INSTRUMENTS.

—1—

LINEAR MEASUREMENTS—LINK-CHAIN.

Equipment: Link-chain, set of marking-pins, two transit-poles, two iron plumb-bobs.

Set a transit-pole at each of two designated hubs about 600 feet apart. Measure the distance *three* times, lining in by eye, taking precautions to have the two ends of the chain always at the same level, "breaking chain" if necessary, etc.

Always drag the chain ahead its full length, even if it is necessary for the head chainman to walk back to "break chain." Always place a pin at each even hundred feet, and begin again at that point for a new chain length.

Each student makes a *set* of three measurements acting as head chainman. The extreme range of the three values should not be more than $\frac{1}{1000}$ of the distance.

NOTES.—For this simple problem a mere description, giving the locality of the work, the three measurements obtained, and other information called for in the general directions, is all that is necessary.

—2—

LINEAR MEASUREMENTS—STEEL TAPE.

Equipment: 100-foot steel tape, two transit-poles, two brass plumb-bobs, foot-rule, hatchet, and twelve stakes.

Set a transit-pole at each of two designated hubs about 600 feet apart. Drive stakes on line, at least one every 100 feet, and also one at every point where it is necessary to "break chain," placing them so that an even-foot mark will come on top of the stake. Measure the distance *three* times. Transfer the measurements to the tops of the stakes very carefully, using a plumb-bob if necessary, and mark the point with a pencil. Measure the final fraction of a foot with the foot-rule. The extreme range of the three values should not be more than $\frac{1}{3500}$ of the distance. Each student makes a set of *three* measurements, acting as head chainman. The intermediate stakes should be pulled and re-driven between each set, so as to make the measurements absolutely independent. The stakes may be aligned *by eye, aided* by the plumb-bobs.

NOTES. [See NOTES for Prob. 1.]

—3—

LINEAR MEASUREMENTS—TESTING CHAIN (OR TAPE).

Equipment: Chain (or tape) to be tested, standard tape, spring-balance, two brass plumb-bobs, triangular scale, and a small strip of cardboard (e.g., visiting-card).

1. Unroll the standard tape in a suitable place.* Set one of the end marks at some well-defined mark on the floor (e.g., a scratch in the top of a nail head) and tack the strip of cardboard under the other end mark of the tape. Pull the tape to a tension of 16 lbs., and mark on the cardboard with a *sharp hard* pencil the position of the 100-foot mark. Make a similar measurement with the chain or tape to be tested, and measure the difference if any.

2. Observe by several trials the changes in the length of the chain (or tape) by making the tension 12 lbs. and then 20 lbs. Compute the average change for one pound variation in pull.

3. Hold the chain (or tape) to be tested clear of the floor and note what tension is necessary to bring the end marks at the same distance apart as when supported throughout its length with a tension of 16 lbs. Use brass plumb-bobs in this test to bring the end marks over the points on the floor.

NOTES. [See NOTES for Prob. 1.]

---

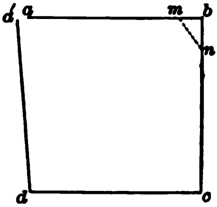
* A passageway or hall is often the only available place, and serves very well except for interference to and by others. A smooth level sidewalk will answer the purpose in fair weather. The vacant space under the seats of a large grand stand may sometimes be utilized as a permanent place for these as well as other long-tape tests.

—4—

CHAIN-SURVEYING—LAYING OUT RIGHT ANGLES.

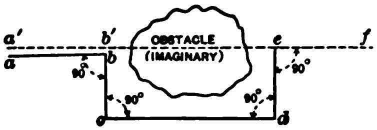
Equipment: Link-chain, set of marking-pins.

1. Select an area where a square about 250 feet on a side can be laid out. Select *a* and *b* *about* 250 feet apart. Lay out a right angle at *b*, using the "3, 4, 5 method." Measure *mb*=30 feet. Fix the zero of the chain at *b* (fastening it with a marking-pin) and have one man hold the 90-foot mark at *m*. Swing the 40-foot mark until both sections of the tape are taut and the angle *mbn* will then be a right angle.



Measure off *bc*=*ab*. Lay out a right angle at *c* and make *cd*=*ab*. Lay out a right angle at *d* and make *da'*=*ab*. Measure *aa'* and compute the "error of closure" (which equals *aa'* divided by the total perimeter).

2. To chain a line through an obstacle (imaginary in this case). Select two points *a* and *b* as in figure. Lay off right angles at *b*, *c*, *d*, and *e*, making *bc*=*de*. Produce *ef* backward and note value (if anything) of *bb'* and *aa'*. Measure *eb*.

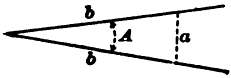


Notes.—Sketches as above with statements of the required results will constitute a sufficient report. The magnitude of the errors will indicate how much reliance can be placed on such methods.

CHAIN-SURVEYING—OBLIQUE ANGLES.

Equipment: Chain, two transit-poles, set of marking-pins.

Set five marking-pins as witnesses near five designated hubs, which should be about 300 feet apart, and form an irregular pentagon.



1. Measure all the angles by the method suggested in the figure, using the formula $\sin \frac{1}{2}A = \frac{a}{2b}$.

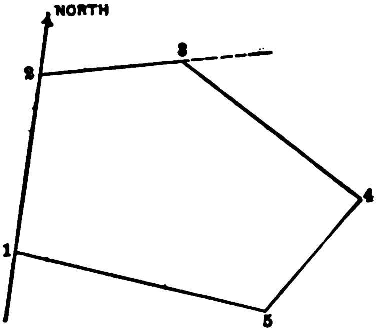
If the angle is much greater than 90°, it may be more convenient to produce one of the sides and measure the supplement of the angle (e.g., angle 3 in the figure). Note whether the sum of all the interior angles is nearly 540°.

Form of Notes.

Angle	b	a	$\frac{a}{2b}$	$\frac{1}{2}A$	A
1	40	58.7	.734	47° 13′	94° 26′
2	40	62.8	.785	51° 43′	103° 26′
3	40	29.5	.369	21° 40′	(sup.) 136° 40′
4	40	55.2	.690	43° 38′	87° 16′
5	40	68.5	.856	58° 52′	117° 44′
					539° 32′

Course.	Bearing. (merid. assumed.)	Distance.	Latitude.	Departure.	
1....2	N	252.8	+ 252.8	——	
2....3	N 76° 34′ E	204.6	+ 47.5	+ 199.0	
3....4	S 60° 06′ E	320.7	− 159.9	+ 278.0	
4... 5	S 32° 38′ W	207.2	− 174.5	− 111.7	
5 ...1	N 85° 06′ W	363.8	+ 31.1	− 362.5	
		1349.1	+ 331.4 − 334.4	+ 477.0 − 474.2	
			3.0	2.8	

2. Measure the sides. Assume one side as meridian, and from the measured angles compute the corresponding bearings of the other sides. For example, the bearing of 3 . . . 4 is S 60° 06′ E; i.e., the bearing of 4 . . . 3 is N 60° 06′ W. The angle at 4 is 87° 16′, which means that 4 . . . 5 runs in a direction 147° 22′ (the sum) from North or 32° 38′ from South. Therefore the bearing of 4 . . . 5 is S 32° 38′ W. Compute the latitudes and departures and then the error of closure. The *latitude* equals the distance times the *cosine* of the bearing; the *departure* equals the distance times the *sine* of the bearing. The *error of closure* is the quotient (generally expressed as a fraction whose numerator is one) of the square root of the sum of the squares of the errors of the latitudes and of the departures divided by the total perimeter.



$$\text{Error of closure} = \frac{\sqrt{3.0^2+2.8^2}}{1349.1} = \frac{1}{329}$$

—6—

COMPASS PRACTICE—SIGHTING AND READING NEEDLE.

Equipment: Surveyor's compass.

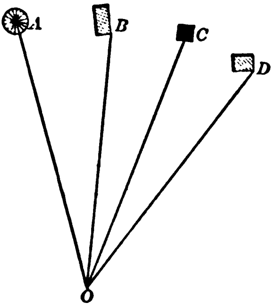
Set up the compass over some fixed hub designated by the instructor in charge and point at four well-defined points (e.g., church steeples, corners of buildings, etc.), which will be also designated. After sighting at all four points in turn and taking the needle reading on each to the nearest 5′ of arc, move the declination arc slightly and again take the needle readings on all four points. Take four such sets of readings. Calling *O* the position of the instrument and *A*, *B*, *C*, and *D* the points sighted at, four values of the angle *AOB* may be obtained by taking the differences of the bearings. Similarly four values of each of the other angles may be obtained. The discrepancies will give some idea of the reliability of needle readings.

Form of Notes.

	1st Reading.	2d Reading.	3d Reading.	4th Reading.	Mean.
Point *A*	N 13° 20′ W	N 15° 10′ W	N 16° 25′ W	N 17° 10′ W	
“ *B*	N 1° 35′ E	N 0° 20′ W	N 1° 30′ W	N 2° 15′ W	
“ *C*	N 64° 30′ E	N 62° 35′ E	N 61° 15′ E	N 60° 45′ E	
“ *D*	N 77° 45′ E	N 76° 0′ E	N 74° 55′ E	N 74° 15′ E	
Angle *AOB*	14° 55′	14° 50′	14° 55′	14° 55′	14° 54′
“ *BOC*	62° 55′	62° 55′	62° 45	63° 00′	62° 54′
“ *COD*	13° 15′	13° 25′	13° 40′	13° 30′	13° 27′

Abnormal discrepancies in the angles may sometimes occur on account of the attraction caused by a knife, keys, or a steel key-chain in the pockets of the observer, or by a steel band in the brim of the observer's hat. A wagon, temporarily halted near the instrument, especially if loaded with iron, will probably cause an appreciable disturbance of the needle. The student will find it a useful exercise, in connection with this problem, to determine the effect, by careful and repeated trials, of purposely allowing such sources of attraction to be held first on one side and then on the other side of the instrument. To be of any value, this test should only be made in a way that might occur *naturally* while using the instrument.

Tower Blind Asylum.
S W. cor. 34th and Market Sts.
Tower of The Bartram.
N. W. cor. Chem. Lab.



— 7 —

COMPASS PRACTICE—ERRORS OF SIGHTING; SETTING BY NEEDLE AND LINING IN.

Equipment: Compass, transit-pole, foot-rule, board about 2 feet long and 6 or 8 inches wide.

1. Error of sighting. Set up the compass where a clear view of 600 feet may be had. Have the rodman pace off 200 feet and lay the board on the ground with its length transverse to the line of sight, taking care to fix the board very firmly. If a bit of smooth clean pavement is at hand, it will be preferable to the board, being rigid. Sight at the rod held at some marked point near the centre of the board. Clamp the vertical axis. Have the rodman measure the distance of this point from one end of the board and then move the

FORM OF NOTES.

Distance.	Distance of rod from end of board.	Error. (d)	d^2		
200	″ 12.0 12.8 12.6 10.8 etc.	″ .18 .62 .42 1.38 etc.	 .0324 .3844 .1764 1.9044 etc.		
Sum = Mean =	121.8 12.18		5.2680		
600	(simi	larly)			
200	(simi	larly)			
600	(simi	larly)			

rod away. Line in the rod again, and again have the rodman measure its position. Repeat at least 10 times. Compute the "probable error" (see pp. (46, 47) of a single sighting in *linear* distance. Reduce this error to its angular value by dividing the error by the distance from the instrument. The quotient is the tangent of the angular error.

Repeat this test at a distance of 600 feet.

2. **Error of setting by needle and lining in.** Read the needle to the nearest 5′ when pointing at some one point on the board. Turn the compass a little on its vertical axis and have the rodman move the rod away. Set the compass at the *same needle reading*, and again line in the rod. Have the rodman note the position of the rod as in the first part of the problem. Repeat 10 times at both 200 and 600 feet distance.

CAUTION.—Be sure that the *declination arc* is not disturbed during the progress of the work.

$$E_1 = .6745\sqrt{\frac{5.268}{(10-1)}} = 0''.52 = 0'.043$$

$$\frac{0.043}{200} = .000215 = \tan 0^\circ\ 0'\ 44''$$

Constant compass reading—N 62° 25′ E

Constant compass reading—N 67° 50′ E

—8—

COMPASS PRACTICE—RUNNING OUT A LINE HAVING A CONSTANT BEARING.

Equipment: Compass, transit-pole, set of marking-pins.

Run out a line about ½ mile long, on uneven ground, setting pins at each station, using a *constant* bearing. Pace the distances between stations. Then run back by the reverse bearings and have the rodman note the discrepancy at each station. After noting the error the rodman transfers the marking pin to the new position, which will make the entire return line continuous and independent. Be careful to select stations where there will be no *local attraction* (e.g., from lamp-posts, iron fences, etc.). Where local attraction is suspected, take a back-sight and determine its amount. The work will demonstrate the compensating character of the errors, the degree of accuracy that may be expected, and the frequency even in suburban localities of greater or less local attraction.

Form of Notes.

Sta.	Bearing.	Distance.	Errors on return.	
0	S 65½° W	350	S 0.4	
1	S 65½° W	420	N 0.6	
2	S 60¼° W	370	N 1.8	Local attr. suspected at sta. 2. B. S. to sta. 1, N 60¼ E.
3	S 65½° W	640	N 0.4	B. S. to sta. 2, N 65½ E.
4	S 65½° W	260	S 1.0	
5	S 65½° W	480	S 0.8	
6	——	370	——	Bearing on return line from sta. 6, N 65½ E.

—9—

LEVEL PRACTICE—READING ROD AND SETTING TARGET.

Equipment: Wye-level, "Philadelphia" level-rod.

1. Send the rodman about 200 feet away on a nearly level stretch. Sight on the rod held on a peg or on a smooth solid knob of stone or brick and direct the setting of the target, which is then read to the nearest .001 foot and recorded by the rodman. The rodman then moves the target a few inches up or down and the target is again set and read. Obtain 10 such values. Find the "probable error" (see pp. (46, 47) of the mean and of a single observation.

2. Turn the rod so that the leveller sees the numbers on the *back* of the rod. The leveller reads the rod directly to the nearest .01 foot and records the reading. The rodman then extends the rod *about* 0.1 foot, reading and recording the exact movement to the nearest .01 foot as shown by the index. The leveller again reads and records the rod reading. This is done 10 times. Subtract the total rod movement as reported afterward by the rodman from the rod reading to which each corresponds. The results should be nearly identical. Compute the "probable error" as in Part 1. This exercise tests the ability to read a "speaking rod."

3. Repeat Part 1 at a distance of about 600 feet. This will test the ability to read a target at a distance.

FORM OF NOTES.

No. of observ.	Rod reading.	d	d²	Dist.		
1	5.203	.001	.000 001	200		
2	5.206	.002	004			
3	5.199	.005	025			
etc.	etc.	etc.	etc.			
...	....	....	....			
10	5.201	.003	.000 009			$E_1 = .6745 \sqrt{\frac{.000\,156}{10-1}} = .0028$
Sum	52.041		.000 156			$E_m = \frac{.0028}{\sqrt{10}} = .00088$
Mean	5.204					
(simi	larly)			600		
No. of observ.	Rod reading.	Rod extension.	Diff.	d	d²	
1	8.63	—	8.63	.02	.0004	Distance about 200 feet.
2	8.74	.08	8.62	.03	9	
3	8.81	.19	8.62	.03	9	
4	8.96	.32	8.66	.01	.0001	
etc.	etc.	etc.	etc.	etc.	etc.	

—10—

LEVEL PRACTICE—DIFFERENTIAL LEVELLING.

Equipment: Wye-level, level-rod.

Run a line of levels over a series of designated hubs. Assume some designated bench-mark to have an elevation of 100 feet above the datum-plane. Record all rod readings when held on turning points for fore-sights in the column headed F. S. Rod readings on points not used as turning-points are recorded as "intermediate sights" (I. S.). The line of levels is to form a complete circuit ending on the hub at which the line started—as a check. Do not use the target on the level-rod. The leveller reads the rod directly, estimating to 100ths of a foot. If a height of over 7 feet is required, the rod is elongated to its full length. (This assumes the use of the "Philadelphia" self-reading rod.) With some rods the target must be used. To guard against error, the target is sometimes used on all turning points (T. P.), the readings taken and recorded independently by *both* rodman and leveller.

FORM OF NOTES.

Sta.	B. S.	H. I.	F. S.	I. S.	Elev.	
B. M.	3.64	103.64			100.00	B. M. on curb at S. E. cor. 34th and Spruce Sts.
T. P.			7.48		96.16	
	4.29	100.45				
Hub$_{25}$				7.19	93.26	
T. P.			7.06		93.39	
etc.						

—11—

LEVEL PRACTICE—PROFILE LEVELLING.

Equipment: Wye-level, level-rod, 50-foot tape. The student must be provided with a small sheet of profile-paper, Plate A.

* Obtain first the standard length of the street-car rails on Woodland Ave. Measure any odd lengths that may be found, and the lengths of frogs, etc., at switches and crossings. Beginning at the first rail-joint on the south side of the south track on Woodland Ave., just west of the switch running on to Walnut St., obtain the elevation of each rail-joint of that line of rails up to the first frog at the intersection of Woodland Ave. and Spruce St.

Assume the datum-plane as 100 feet below the curb at the S.W. corner of 34th and Walnut Sts. Note the elevation of the curb at the east intersection of Woodland Ave. and Spruce St.

Draw the profile on profile-paper (Plate A), horizontal scale 100 feet to the inch, vertical scale 10 feet to the inch.

NOTES.—Similar to those for Prob. 10. Rod readings for rail-joints should be put in column "I. S." except when used as turning-points. The "stations" in the first column should be in each case the continued sum of all the rail-lengths up to that point, e.g., 0, 0 + 30, 0 + 60, 0 + 90, 1 + 20, etc. As a check on the numerical work, note whether the sum of the back-sights (B. S.) minus the sum of the fore-sights (F. S.) equals the elevation of the first bench-mark minus the elevation of the final bench-mark.

---

* It was considered that these directions could be made most valuable by applying them to a definite locality rather than to make them purely general. The necessary modifications to apply them to any locality can be readily made. These remarks apply equally to several other problems.

—12—

LEVEL PRACTICE—TESTING ADJUSTMENTS.

Equipment: Wye-level, level-rod, 100-foot tape, set of marking-pins.

The object of this problem is to make the student as familiar as possible with the adjustments, without subjecting the instrument to the injury that would result from excessive disturbance of the adjusting-screws, especially in unskilful hands. Therefore no adjusting-screws are to be disturbed, but the value of each error of adjustment is determined, assuming all previous adjustments as already made. Making three trials of each adjustment will demonstrate to the student that unless *extreme care* is taken in handling the instrument, the three trials will not exactly agree, and that he will sometimes find an apparent small error when the adjustment is practically perfect. This note applies equally to Problem 16.

1. To make the line of sight parallel to the axis of the bubble.

Form of Notes.

	1st Trial.	2d Trial.	3d Trial.	Mean.	
1st Adj. (*a*)					
Tel. dir.	3.684	3.986	3.902		
Tel. inv.	3.692	3.995	3.909		
½ diff.	.004	.0045	.0035	.004	
1*b*	0.6 div.	0.8 div.	0.7 div.	0.7 div.	
2					
3	0.8 div.	1.1 div.	0.9 div.	0.9 div.	
Ang. val.	of one div. of	bubble			
E	15.8 3.6 12.2	15.4 5.6 9.8	14.8 4.2 10.6		
O	15.2 2.8 12.4	13.2 3.1 10.1	13.1 2.2 10.9		
R	4.931 4.211 .720	4.867 4.294 .573	4.831 4.208 .623		
a″	30″.2	29″.7	29″.9	29″.93	

a. Measure off a base 400 feet long on nearly level ground and set up the level at one end of the line. Sight at the rod at the other end. Have the vertical axis clamped, and the whole instrument as rigid as possible except the clips over the telescope, which should be loose so that the telescope can be rotated in the wyes without shaking the instrument. If the line of collimation is in adjustment the rod reading will be the same, when the telescope is rotated half-way, as at first. The difference, if any, is *twice* the error. ***Make three tests.*** Raise or lower the telescope between each test so as to bring the readings on a different part of the rod.

b. Bring the telescope directly over a pair of levelling-screws and clamp the vertical axis. Bring the bubble to the centre with the levelling-screws. Reverse the telescope in the wyes. If the bubble again comes to the centre, the wyes are level and the line of collimation is parallel to the axis of the bubble. If the bubble does not come to the centre, move the levelling-screws until the bubble is moved *half-way* back. Reverse the telescope again, and the bubble will come very nearly if not quite to its position before reversal. Keep adjusting the levelling-screws until the bubble remains in one

dist. = 400 ft.

Bubble moves forward about one division
when swung to the right about 30°;
moves backward when swung to the left.

dist. = 400 ft.

place in the tube for the telescope in either position. Under those conditions the wyes are level and distance of the centre of the bubble from the centre of the tube is the error of adjustment. Record this distance measured in divisions of the bubble-tube. ***Make three tests.***

2. To bring the bubble axis into the vertical plane through the axis of the telescope. Bring the bubble to the centre of the tube, first observing that the bubble-tube hangs directly under the telescope, Rotate the telescope slightly in the wyes, and note whether the bubble remains in the centre, or if it moves in one direction for a swing to the right and in the other direction for a swing to the left.

3. To make the axis of the wyes perpendicular to the vertical axis of the instrument. Bring the bubble to the centre over both pairs of levelling-screws and then rotate the instrument about its vertical axis 180°. Correct one half the error, if any, with the levelling-screws; make the same correction for the other levelling-screws. The bubble will then remain nearly, if not quite, in the same place for any position of the telescope. Keep adjusting until this is true. The distance of the centre of the bubble from the centre of the tube is a measure of the correction that should be made by the adjusting-screw under one of the wyes. Record the error, if any, in divisions of the bubble-tube. ***Make three tests*** by purposely throwing the instrument out of level after each test and making an independent determination.

Value in seconds of arc of one division of the level-tube. Run the bubble to near one end of the tube, but still keeping both ends of the bubble within the graduated divisions on the tube. Take a reading on the rod held 400 feet away. Move the bubble to the other end of the tube and again read the rod.

Let E = the movement, in divisions, of the eye end of the bubble;

Let O = the movement, in divisions, of the object end of the bubble;

Let R = difference in rod readings;

$$\text{Let } \alpha'' = \frac{R}{400\,\frac{E+O}{2}\,\sin 1''}.$$

Obtain three values.

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TRANSIT PRACTICE—ANGLE MEASUREMENTS.

Equipment: Transit.

Set up the transit over a designated hub and point at two well-defined points (edge of a cornice seen against the sky or a church spire) which will be also designated. With vernier A at *any* reading (i.e., *not* 0°) set the vertical cross hair on the *left hand* point. Read the degrees and minutes of vernier A and the *minutes* of vernier B. Take the mean of the readings of minutes. That, with the degrees of vernier A, constitutes the "mean." Loosen the upper plate, point at the other mark, and clamp again. Again take the mean reading of the verniers. The difference of the means is the horizontal angle. With the upper plate clamped and lower plate loose, turn back to and sight on the left-hand mark and clamp the lower plate. Loosen the upper plate; point at the right-hand mark; clamp the upper plate; take the mean of the vernier readings. The difference between this mean and the previous mean should be practically the same as before. Continue this method until *ten* such values of the angle are obtained. They should not differ more than 1′ of arc. Estimate each vernier reading to half-minutes. Compute the "probable error" of the mean and of a single value of the angle.

Form of Notes.

Vern. A.	Vern. B.	Mean.	Diff.	d	d^2	
20° 16′ 0″	15′ 30″	20° 15′ 45″	——	——	——	Left-hand mark, church steeple, cor. A. & B. Sts.
74° 18′	18′	74° 18′	54° 02′ 15″	6″	36	Right-hand mark, tower finial, cor. M. & N. Sts.
128° 20′ 30″	21′	128° 20′ 45″	54° 01′ 45″	24	576	
182° 22′	22′ 30″	182° 22′ 15″	54° 01′ 30″	39	1521	
236° 24′ 30″	24′	236° 24′ 15″	54° 02′	9	81	$E_1 = .6745\sqrt{\frac{5264}{10-1}} = 16''.3$
etc.	etc.	etc.	etc. ...	etc. ..	etc. ...	$Em = \frac{16.3}{\sqrt{10}} = 5''.1$
		Mean	21′ 30″ 54° 02′ 09″		5264	

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TRANSIT PRACTICE—TRAVERSING.

Equipment: Transit, transit pole, set of marking-pins.

Set five marking-pins as witnesses near five designated hubs. Set up the transit over one hub; set vernier A at zero; loosen the needle and revolve the whole instrument until the telescope points south, as shown by the needle. Clamp the lower plate; loosen the upper plate and point at the next hub; read vernier A and the needle. Set up the transit at the next hub; plunge the telescope and point at the previous station; test vernier A to see that the plate has not slipped; read the needle; plunge the telescope again to direct position and turn to the third hub and again read vernier A and needle. Take observations similarly at hubs 3, 4, and 5; when at station 5 take a fore-sight to station 1. Finally, reoccupy station 1, sighting back at station 5 for a back-sight; the reading of vernier A for the fore-sight to station 2 should exactly agree with the former reading. A difference of more than a very few minutes of arc indicates gross inaccuracy or a blunder. The object of the needle readings is to guard against blunders. As soon as the vernier and needle readings are recorded, note whether the azimuth and needle reading agree. For example, an azimuth of 217° 42′ signifies a bearing of N 37° 42′ E, and the needle reading should be as near this as it may be read, unless there is local attraction. The degrees on the horizontal plate should be read from the row that is continuous to 360°.

FORM OF NOTES.

Inst. at	Pointing to	Vern. A	Needle.			
Sta. 1	South	0°	South			
	△2	274° 03′	S 86° E			
△2	△1	274° 03′	S 85° 50′ E		Tel. rev.	
	△3	192° 16′	N 12° 20′ E			
△3	△2	....	....		Tel. rev.	
	△4	....	....			
etc.						

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TRANSIT PRACTICE—ROD ALIGNMENT; SETTING ALIDADE.

Equipment: Transit, transit-pole, board about two feet long and six or eight inches wide, foot-rule.

1. Send the rodman to a point about 300 feet distant; have him lay the board on the ground where it will be firm, with its length perpendicular to a line to the transit; have him set the rod on the board (near the middle) and measure and record the distance to one end of the board (measuring in inches and *tenths*). The transitman sights at the rod, clamping both plates. The rodman moves the rod away and is again lined in by the transitman. The rodman again measures and records the distance to the end of the board. Obtain 10 such values. Compute the "probable error" of a single sighting and of the mean of all the sightings. Reduce the probable error of the mean to its angular value $\left(\frac{x}{300} = \tan \alpha\right)$.

2. The transitman reads vernier A when the transit points at some one setting of the rod, and between each subsequent setting unclamps and moves slightly the alidade, then resets the alidade at the *same reading* and again lines in the rod as in Part 1. Obtain 10 such values. Compute the probable error and the angular value of the error as before (Part 1).

3. Repeat both of these tests at a distance of about 1000 feet, which may be measured by pacing.

Form of Notes.

Dist.	No. of Rod Setting.	Dist. of Rod from End of Board.	d	d^2		
300	1	12.2	.04	.0016		
	2	12.5	.34	.1156		
	3	12.7	.54	.2916		$E_1 = .6745 \sqrt{\frac{1.6240}{10-1}} = .286''$
	etc.	etc.	etc.	etc.		$= .024'$
	....	...	..	...		
	10	11.8	.36	.1296		$Em = \frac{.286}{\sqrt{10}} = .090'' = .0075'$
		121.6		1.6240		$\frac{.0075}{300} = .000025 = \tan 0^\circ\, 0'\, 05''$
	Mean	12.16				
	1	....	....	...	Vern. A	
300	2	....	....	..	30° 27'	

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TRANSIT PRACTICE—TESTING ADJUSTMENTS.*

Equipment: Transit, transit-pole, level-rod, 100′ steel-tape, foot-rule.

1. **To make the plane of the plate bubbles perpendicular to the vertical axis.** Set up the transit and make the plate bubbles parallel to opposite levelling-screws. Bring both bubbles to the centre; revolve exactly 180°. The distance (measured in divisions on the bubble-tube) of the *centre* of the bubble from the *centre* of the scale is *twice* the error. With the levelling-screws throw the transit out of level and begin over again so as to make an absolutely independent redetermination. *Obtain three independent values for the error of this adjustment.*

2. **To make the line of sight perpendicular to the horizontal axis of the telescope.** Set the transit where a clear sight of about 300 feet each way in opposite directions may be obtained. Point the telescope at some well-defined point (e.g., the corner of a building having a good background). Plunge the telescope and line in the rodman at some point about 300 feet away. Revolve the alidade and point at the first point selected. Again plunge the telescope and line in the rodman for a point beside the one previously set. The distance between these two points is *four* times the error. *Make three tests.*

3. **To make the horizontal axis of the telescope perpendicular to the vertical axis of the transit.** Set up the transit about 20 feet from the vertical wall of a building. Select a point about as high on the wall as the telescope may be made to point at. Swing the telescope down and have the rodman make a mark on the wall down near the ground. Plunge the telescope, revolve the instrument 180°, and again point at the upper point. Swing the telescope down and have the rodman make another mark beside the first lower mark. A vertical plane through the instrument and the upper mark will pass midway between the two lower marks. Record the distance of the instrument from the wall, the vertical distance (estimated) between the upper and lower points and the distance between the two lower points. *Make three tests.*

4. **To make the axis of the telescope bubble parallel to the line of sight.** Level the telescope and mark a point on the ground directly under the eyepiece (driving a peg if necessary). Measure

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* *See first paragraph* of Problem 12, page 16.

the height of the centre of the eyepiece above the point with the level-rod; call the reading *a*. Establish another point about 300 feet away and at about the same level. Read the level-rod placed on it; call the reading *b*. Transfer the instrument to the second point and set it up so that the eyepiece is directly over the point. Measure the height of the eye-piece above the point, calling it *c*, and also read the level-rod held on the first point, calling it *d*. Then we should have $d - a = c - b$ if the instrument is in adjustment. If not, then $(d - a) - (c - b) = 2m$. $\frac{m}{\text{distance}} = \tan \alpha$. If *m* is positive the line of collimation points upward. *Make three tests.*

5. To make the vernier of the vertical circle read zero when the line of sight is horizontal. This test is only applicable to instruments having fixed vertical circles. As an adjustment it is only applicable to instruments having adjustable verniers. Make the telescope level by means of the level bubble and note the *amount* and *direction* of the "index error." *Make three tests* by turning the telescope out of level between each test.

Best location for these tests. Tests 2 and 4 require an unobstructed, nearly level stretch of 300 feet in one direction, and Test 2 a suitable point about 300 feet in the opposite direction. Test 3 requires a high wall about 20 feet from the instrument, and Tests 1 and 5 may be made anywhere. When an unobstructed line of sufficient length can be found, parallel to a wall and about 20 feet away, a single setting of the instrument will serve for all the tests.

FORM OF NOTES.

Test.	1st Trial.	2d Trial.	3d Trial.	Mean.		
1	N .3 div. E 1.2 "	N .5 div. E 1.0 "	N .4 div. E 1.3 "	N .40 div. E 1.17 "		Bub. par. to tel. " perp. " "
2	1½″	1¼″	1⅝″	1″.46	$\frac{1.46}{4} = 0.365$	dist. about 350 ft.
3	$\frac{5}{16}$″	⅜″	¼″	$\frac{5}{16}$″	$\frac{5}{16} \div 2 = \frac{5}{32}$	wall 24 ft. from transit; upper and lower points about 40 ft. apart.
4 *a*	4.26	4.51	4.51		dist. = 364	
b	3.78	4.04	4.02			
c	4.61	4.61	4.47			
d	5.13	5.14	5.03			
m	.02	.03	.035			
tan α	.000 054	.000 081	.000 094			
α	+0° 0′ 11″	+0° 0′ 16″	+0° 0′ 19″	+0° 0′ 15″		
5	+0° 3′	+0° 3½′	+0° 2½′	+0° 03′		

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TRANSIT PRACTICE—STADIA READINGS.

Equipment: Transit, stadia-rod, 5-foot rod (a light rod, graduated in feet and tenths, for obtaining H. I).

Send the rodman to a point about 300 feet away. Note the height of the horizontal axis of the telescope above the ground. Measure the "$f+c$." Point at the rod with the middle cross-wire at a reading on the rod equal to the height of the axis. Read the upper, middle, and lower cross-hairs, also the vertical arc. Note the equality of the upper and lower wire interval. For a check on the distance and as additional practice, move the tangent-screw to the vertical arc *very slightly*, and reread the three cross-hairs. Take *six* such readings. [Although by strict theory these wire intervals should differ slightly with any change in the vertical angle, there should be no appreciable variation in them for slight changes, especially if the vertical angle itself is small. For example, with a vertical angle of 10° and a distance of 1000 feet, the vertical angle would have to be changed 10' in arc to make a difference of 1 foot in the apparent horizontal distance, and 10' of arc in 1000 feet would mean a movement of 2.9 feet of the wire over the rod.] Compute the true horizontal distance and the difference of elevation.

Repeat these observations at about 600 feet and at about 1000 feet.

FORM OF NOTES.

Approx. Dist.	Wire Readings.	Vertical Angle.	Horizontal Distance.	Diff. Elev.		
300	6.24 4.80 3.36	- 2° 24'	289	- 12.0'		H. I = 4.80 $f+c = 1.6$
300	6.27 4.83 3.38					
etc.	etc.					

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TRANSIT PRACTICE—TRAVERSING WITH STADIA.

Equipment: Transit, stadia-rod, set of marking-pins, 5-foot rod.

Make a traverse as in Problem 14, the distances between stations being measured by stadia both while taking fore-sights and back-sights. Unless the transit has a full vertical circle the vertical angle for the back-sight cannot be read with the telescope plunged. It is therefore necessary with most instruments when taking a back-sight to set vernier A at a reading exactly 180° different from the reading for fore-sight, leaving the telescope in direct position. The needle reading should also be 180° different from its reading for fore-sight. Always direct the middle wire to a point on the rod at the same height as H. I. to obtain the vertical angle. The horizontal distance may be most quickly read by raising or lowering the telescope until the lower cross-wire comes on the nearest even footmark and then reading the upper wire. As shown in the previous problem, this involves no appreciable error. Check the horizontal angles by the closure of the last course. Also see whether the algebraic sum of the differences of elevation equals zero. Compute the true horizontal distance to the nearest even foot, the vertical difference of elevation to the nearest even tenth of a foot.

FORM OF NOTES.

Sta.	Vern. A	Needle.	Vert. Ang.	Wire Interval.	Hor. Dist.	Diff Elev.	
Inst. at △1	0°	S					
△2	20° 16′	S 20° 15′ W	− 2° 16′	1.80			
Inst. at △2					181	− 7.1	
B. S. △1	200° 16′	N 20° 15′ E	− 2° 15	1.80			
△3	118° 47′	N 61° 10′ W	+ 1° 22′	1.18			
Inst. at △3							
etc.							

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TRANSIT PRACTICE—SAEGMULLER SOLAR ATTACHMENT.

Equipment: Transit with solar attachment, declination tables. The vertical circle of the transit should be fixed and have a vernier reading to half-minutes.

Compute the declination settings for each half-hour for the total period of observations. Set up the instrument over a designated hub. Level up with *extreme care* and note the index error (if any) of the vertical circle. Set vernier A at zero and point at another designated hub or well-defined point. Loosen the upper plate and swing into the meridian by means of an observation on the sun. Read

FORM OF NOTES.

Time.	Decl.	Refr. Corr.	App. Decl.	Azimuth.	Needle.
2:30	+10° 27′	+0′ 42″	10° 27½′	15° 06′	N 7° 15′ W
36				05	
42				08	
51				05	
3:00	+10° 27½′	+0′ 48″	10° 28′	06	
etc.				etc.	
				....	
				63	
			Mean	15° 06.3′	

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PLANE-TABLE PRACTICE.

Equipment: Plane table (including alidade, table-level, and compass-box), stadia-rod, steel tape, set of marking-pins. Students must each provide a sheet of drawing-paper about 16″ × 24″ (detail paper will answer the purpose), scale, *hard sharp* pencil, and rubber.

Measure a base-line (*AB*) about 300 feet long. Set up the plane table at *A*. Orient the table and draw in the base-line (30 feet to an inch) so that the proposed sketch will come properly on the paper. Draw a magnetic meridian through *A*, using the compass-box. Draw lines toward several points (trees, building corners, etc.), which are visible from both *A* and *B* and, determining their distances by stadia, plot them at once to scale.

Set up the table at *B*; orient by sighting to *A*; check the orientation *with the compass*; check the plotting of the other points by vernier A. Then loosen the upper plate and swing out of the meridian. Swing into the meridian again by means of another observation and again read azimuth by vernier A. Take 10 such observations and compute the probable error of the mean and of a single observation. Observe the azimuth (0°) of the other mark occasionally to see that the lower plate has not slipped. The mean reading of vernier A, if the angles are read in the direction S, E, N, W, will be the azimuth of the line between the hubs. The observations should be taken as rapidly as possible, the change in apparent declination being so small during half an hour that interpolations may be used.

Observe the needle reading when the instrument is in the meridian; it should be the declination of the needle for the time and place.

d	d^2	
0.3	.09	$E_1 = .6745\sqrt{\frac{15.90}{10-1}} = .99' = 59''$ $E_m = \frac{.99}{\sqrt{10}} = .31' = 18''.6$
1.3	1.69	
1.7	2.89	
1.3	1.69	
0.3	.09	
etc.	etc.	
....	....	
....	15.90	

(PLANE-TABLE PRACTICE—*continued.*)

sighting to them. Sight to some station point *C*, visible from both *A* and *B*, drawing an indefinite line toward it.

Set up the table at *C*; orient by sighting to *B*; check the orientation with the compass; locate *C* on the drawing by sighting to *A*. Check the plotting of *C* by sighting at some of the other points already located.

NOTES.—In the note-book record date, time of work, number of problem, and personnel of party. The drawing, properly lettered, should be handed in with the note-book. Each student of the party may plot *his own* sheet for each station while the instrument is set there—thus saving time. When another sheet is placed on the table at any station, it simply requires a readjustment of the orientation. *The accuracy is* not necessarily impaired. Extreme accuracy in *drawing is essential* for good results.

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SEXTANT PRACTICE.

Equipment: Sextant, artificial horizon.

1. Horizontal Angles. Select four well-defined points at least ½ mile away (e.g., chimneys, steeples, etc.). Obtain *four* values for each of the six angles formed at the instrument by the points taken in pairs, as shown below. Loosen the clamp and move the arm slightly between each reading. Unless the points sighted at are very far away, it will be necessary to use considerable care in holding the sextant over a fixed point or an appreciable variation in the angle may be the result.

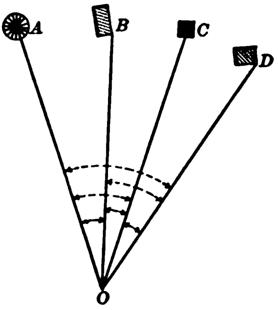
2. Vertical Angles. Set the artificial horizon on a rigid base

Form of Notes.

Angle.	Readings.	Mean.			
AOB	12° 16′ 20″				
	10″				
	30″				
	20″	12° 16′ 20″			
BOC	14° 22′ 40″				
	etc.				
(similarly)					
COD					
AOC					
BOD					
AOD					
Time.	**Double Altitude.**	**Altitude.**			
2 22 35	82° 37′ 10″	41° 18′ 35″			
28 45	etc.	etc.			
31 15	....	....			

and take a position so that the sun will be reflected from the mercury to the instrument. Observe the double altitude of the "upper limb" of the sun by bringing the index image of the sun just tangent to the direct reflection seen through the telescope in the mercury and *apparently* on the *under* side of the mercury image. One half of the angle, minus the refraction correction, minus the semi-diameter of the sun, gives the altitude of the sun. As soon as the images are brought into contact, note the exact time and then read the angle. Obtain 10 values for the altitude of the sun. During the afternoon these values will be constantly decreasing—during the morning they will be increasing. Unfortunately there is no check on this work except a calculation of what the altitudes should be at the given times, which would probably not be obtained closely enough for a very good check.

A; Church steeple, cor. E. and F. Sts.
B; N. E. cor. G. and H. Sts.
C; Tower, City Hall.
D; S. W. cor. M. and N. Sts.



Upper limb of the sun.

(c) Raise or lower the slide near the top
he side until a horizontal plane through
ls just tangent to the upper surface of the
column. (d) Read the vernier; the small-
ld thousandths may easily be estimated if
te the effect of temperature on the height
the readings to 32° F.," by means of the
interpolating when necessary. For exam-
29".632 and 74°; interpolating between 29.5
ain a correction of .121, which is always *sub-*
f values given in the table. 29.632 — .121 =
eferable to hang the mercurial barometer in a
n sheltered from the sun, it may have the same
xternal air. If this is inconvenient, a ther-
placed immediately outside the office (sheltered
readings taken when the barometer is read. The
" should be determined from the *attached* ther-
correction for temperature should be computed
s of the external thermometer and of the thermom-
ing the aneroid. (g) Plot the readings immediately
(Plate A), using time as abscissæ (one hour per inch)
barometric readings as ordinates (.040" of mercury per
curve of pressure is very irregular, it indicates an un-
ition of the atmosphere, or, possibly, very inaccurate

of Ex- nal Air.	Approx. Field Read.	Approx. Office Read.	Diff.	Corr. for Temp.	Diff. Elev.
	308	253	+ 55	+ 2	+ 57
	218	256	− 38	− (+ 2)	− 40
	203	256	− 53	− (+ 2)	− 55

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BAROMETRIC LEVELLING

Equipment: Mercurial and aneroid barometers, pocket thermometer. Each student must be provided with a watch, which should be compared before beginning work, and a small sheet of profile-paper (Plate A).

Use of the Mercurial Barometer. The mercurial barometer is to be observed in its fixed position in the office. (*a*) Read the attached thermometer; this should be done first, as the heat of the observer's body will be sufficient to alter the reading in a short time. (*b*) Bring the surface of the mercury in the cistern to the tip of the ivory index. This is most accurately done by raising the mercury (by means of the large screw at the bottom) until the tip dips slightly into the mercury and forms a slight depression; lower the mercury until, as seen by the reflection of light on the surface of the mercury, the depression just disappears. A magnifier will be of assistance in doing this. Tap the barometer *very gently* to destroy the adherence

FORM OF NOTES.

MERCURIAL BAROMETER READINGS.

Time.	Barom.	Att'd Therm.	Red. to 32°.	Temp. of External Air.	Corr. Reading.
2:00 P.M.	29.846	72°	− .117	68°	29.729
2:15	29.839	71	− .114	68	29.725
etc.					

ANEROID BAROMETER READINGS.

Time.	Place.	Aneroid.	Therm.	Corr. Aner.	Corr. Merc.
2:10	Office	29″.582	74°	——	29″.726
2:25	Roof	.518	62	29.668	.723
2:40	Bridge	.616	65	.761	.720
2:50	Dock	.632	65	.777	.720
3:05	Office	.576	72	——	.722

of the mercury to the glass. (*c*) Raise or lower the slide near the top by means of the screw on the side until a horizontal plane through the lower edge of the slide is just tangent to the upper surface of the *meniscus* at the top of the column. (*d*) Read the vernier; the smallest reading is .002″, but odd thousandths may easily be estimated if necessary. (*e*) To eliminate the effect of temperature on the height of the column, "reduce the readings to 32° F.," by means of the accompanying Table I, interpolating when necessary. For example, assume readings of 29″.632 and 74°; interpolating between 29.5 and 30.0 for 74° we obtain a correction of .121, which is always *subtractive* for the range of values given in the table. 29.632 — .121 = 29.511. (*f*) It is preferable to hang the mercurial barometer in a place where, although sheltered from the sun, it may have the same temperature as the *external* air. If this is inconvenient, a thermometer should be placed immediately outside the office (sheltered from the sun) and readings taken when the barometer is read. The "reduction to 32°" should be determined from the *attached* thermometer, but the correction for temperature should be computed from the readings of the external thermometer and of the thermometer accompanying the aneroid. (*g*) Plot the readings immediately on profile-paper (Plate A), using time as abscissæ (one hour per inch) and reduced barometric readings as ordinates (.040″ of mercury per inch). If the curve of pressure is very irregular, it indicates an unstable condition of the atmosphere, or, possibly, very inaccurate

Temp. of External Air.	Approx. Field Read.	Approx. Office Read.	Diff.	Corr. for Temp.	Diff. Elev.
63°	308	253	+ 55	+ 2	+ 57
62	218	256	− 38	− (+ 2)	− 40
60	203	256	− 53	− (+ 2)	− 55

readings of the barometer. In either case, the computed elevations will be very unreliable.

Use of the Aneroid. Handle the aneroid with *extreme care*, being especially careful that it is never violently jarred by concussion. Always allow it to lie flat when taking a reading. Tap it with the finger very gently before taking the reading. Ignore the thermometer inside of the aneroid and observe instead the pocket thermometer. If this has been carried in the pocket, allow it to remain in the air (shaded from the sun) long enough to indicate the true temperature of the air. Take readings* (1) beside the mercurial barometer; (2) on the roof of the College Building above the Society rooms; (3) on the floor of the South St. bridge; (4) on the edge of the dock immediately underneath the bridge; (5) finally, another reading beside the mercurial barometer. Correct the aneroid readings on the roof, bridge, and dock by the mean of the discrepancies between the aneroid readings in the office and the reduced mercurial readings. Draw a curve through the ordinates of pressures and scale off the amounts of the ordinates for the times when the aneroid readings were taken on the roof, bridge, and dock. Interpolate for temperature at these times. With these reduced and computed values for the pressures and temperatures, the differences of elevation may be computed from Tables II and III. For example, the calculation of the difference of elevation of "Office" and "Roof" is as follows:

From Table II, for 29.663, we have 366 − (6.8 × 9.2) = 308
Similarly, " 29.723, " " 274 − (2.8 × 9.2) = 253

Approx. diff. of elevation........................ 55 feet.

62° + 63° = 125°. From Table III the coefficient for correction is .0262 + (5 × .00106) = .0315. For this and similar cases where the difference of elevation is small, it is only necessary to obtain from the table an approximate coefficient (e.g., .03 in this case) by mere inspection. .03 × 55 = 2, the correction, obtained only to the nearest foot. The correction is *positive*, hence the difference of elevation is 57 feet.

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* The directions are here purely local in their application, but were introduced as being suggestive of methods of obtaining a total difference of elevation of 100 feet or over, which is sufficient to illustrate the use of these instruments.

TABLE I.—REDUCTION OF BAROMETER READING TO 32° F.

Temp	Inches.										
	26.0	26.5	27.0	27.5	28.0	28.5	29.0	29.5	30.0	30.5	31.0
45	−.039	−.039	−.040	−.041	−.042	−.042	−.043	−.044	−.045	−.045	−.046
46	.041	.042	.043	.043	.044	.045	.046	.046	.047	.048	.049
47	.043	.044	.045	.046	.047	.048	.048	.049	.050	.051	.052
48	.046	.047	.047	.048	.049	.050	.051	.062	.053	.053	.054
49	.048	.049	.050	.051	.052	.052	.054	.054	.055	.056	.057
50	.050	.051	.052	.053	.054	.055	.056	.057	.058	.059	.060
51	.053	.054	.055	.056	.057	.058	.059	.060	.061	.062	.063
52	.055	.056	.057	.058	.059	.060	.061	.062	.064	.065	.066
53	.057	.058	.060	.061	.062	.063	.064	.065	.066	.067	.068
54	.060	.061	.062	.063	.064	.065	.067	.068	.069	.070	.071
55	.062	.063	.064	.065	.066	.068	.069	.070	.071	.073	.074
56	.064	.065	.067	.068	.069	.070	.072	.073	.074	.075	.077
57	.067	.068	.069	.070	.072	.073	.075	.076	.077	.078	.080
58	.069	.070	071	.073	.074	.076	.077	.078	.080	.081	.082
59	.072	.073	.074	.075	.077	.078	.080	.081	.083	.084	.085
60	.074	.076	.077	.078	.079	.081	.082	.084	.085	.086	.088
61	.076	.077	.079	.080	.082	.083	.085	.086	.088	.089	.091
62	.079	.080	.082	.083	.085	.086	.088	.089	.091	.092	.094
63	.081	.082	.084	.085	.087	.088	.090	.091	.093	.095	.096
64	.083	.085	.086	.088	.090	.091	.093	.094	.096	.097	.099
65	.086	.087	.089	.090	.092	.093	.095	.097	.099	.100	.102
66	.088	.089	.091	.093	.095	.096	.098	.099	.101	.103	.105
67	.090	.092	.094	.095	.097	.099	.101	.102	.104	.106	.108
68	.093	.094	.096	.098	.100	.101	.103	.105	.107	.108	.110
69	.095	.097	.099	.100	.102	.104	.106	.107	.110	.111	.113
70	.097	.099	.101	.103	.105	.106	.109	.110	.112	.114	.116
71	.100	.101	.103	.105	.107	.109	.111	.113	.115	.117	.119
72	.102	.104	.106	.108	.110	.112	.114	.116	.118	.120	.122
73	.104	.106	.108	.110	112	.114	.116	.118	.120	.122	.124
74	.107	.109	.111	.113	.115	.117	.119	.121	.123	.125	.127
75	.109	.111	.113	.115	.117	.119	.122	.124	.126	.128	.130
76	.111	.113	.116	.118	.120	.122	.124	.126	.128	.130	.133
77	.114	.116	.118	.120	.122	.124	.127	.129	.131	.133	.136
78	.116	.118	.120	.122	.125	.127	.129	.131	.134	.136	.138
79	.118	.120	.123	.125	.127	.129	.132	.134	.137	.139	.141
80	.121	.123	.125	.127	.130	.132	.135	.137	.139	.141	.144
81	.123	.125	.128	.130	.132	.134	.137	.139	.142	.144	.147
82	.125	.128	.130	.132	.135	.137	.140	.142	.145	.147	.149
83	.128	.130	.133	.135	.138	.140	.142	.145	.147	.149	.152
84	.130	.132	.135	.138	.140	.142	.145	.147	.150	.152	.155
85	.132	.134	.137	.140	.143	.145	.148	.150	.153	.155	.158
86	.135	.137	.140	.142	.145	.148	.150	.153	.155	.158	.161
87	.137	.139	.142	.144	.148	.150	.153	.155	.158	.161	.163
88	.139	.142	.145	.147	.150	.152	.155	.158	.161	.163	.166
89	.142	.144	.147	.150	.153	.155	.158	.161	.164	.166	.169
90	.144	.147	.150	.153	.155	.158	.161	.164	.166	.169	.172
91	−.146	−.149	−.152	−.155	−.158	−.160	−.163	−.166	−.169	−.172	−.175

TABLE II.—BAROMETRIC ELEVATIONS.*

B	*A*	Diff. for .01.	*B*	*A*	Diff. for .01.	*B*	*A*	Diff. for .01.
Inches.	Feet.	Feet.	Inches.	Feet.	Feet.	Inches.	Feet.	Feet.
20.0	11,047	− 13.6	23.7	6,423	− 11.5	27.4	2,470	− 9.9
20.1	10,911	13.5	23.8	6,308	11.4	27.5	2,371	9.9
20.2	10,776	13.4	23.9	6,194	11.4	27.6	2,272	9.9
20.3	10,642	13.4	24.0	6,080	11.3	27.7	2,173	9.8
20.4	10,508	13.3	24.1	5,967	11.3	27.8	2,075	9.8
20.5	10,375	13.3	24.2	5,854	11.3	27.9	1,977	9.7
20.6	10,242	13.2	24.3	5,741	11.2	28.0	1,880	9.7
20.7	10,110	13.1	24.4	5,629	11.1	28.1	1,783	9.7
20.8	9,979	13.1	24.5	5,518	11.1	28.2	1,686	9.7
20.9	9,848	13.0	24.6	5,407	11.1	28.3	1,589	9.6
21.0	9,718	12.9	24.7	5,296	11.0	28.4	1,493	9.6
21.1	9,589	12.9	24.8	5,186	10.9	28.5	1,397	9.5
21.2	9,460	12.8	24.9	5,077	10.9	28.6	1,302	9.5
21.3	9,332	12.8	25.0	4,968	10.9	28.7	1,207	9.5
21.4	9,204	12.7	25.1	4,859	10.8	28.8	1,112	9.4
21.5	9,077	12.6	25.2	4,751	10.8	28.9	1,018	9.4
21.6	8,951	12.6	25.3	4,643	10.8	29.0	924	9.4
21.7	8,825	12.5	25.4	4,535	10.7	29.1	830	9.4
21.8	8,700	12.5	25.5	4,428	10.7	29.2	736	9.3
21.9	8,575	12.4	25.6	4,321	10.6	29.3	643	9.3
22.0	8,451	12.4	25.7	4,215	10.6	29.4	550	9.2
22.1	8,327	12.3	25.8	4,109	10.5	29.5	458	9.2
22.2	8,204	12.2	25.9	4,004	10.5	29.6	366	9.2
22.3	8,082	12.2	26.0	3,899	10.4	29.7	274	9.2
22.4	7,960	12.2	26.1	3,794	10.4	29.8	182	9.1
22.5	7,838	12.1	26.2	3,690	10.4	29.9	91	9.1
22.6	7,717	12.0	26.3	3,586	10.3	30.0	0	9.1
22.7	7,597	12.0	26.4	3,483	10.3	30.1	− 91	9.0
22.8	7,477	11.9	26.5	3,380	10.3	30.2	181	9.0
22.9	7,358	11.9	26.6	3,277	10.2	30.3	271	9.0
23.0	7,239	11.8	26.7	3,175	10.2	30.4	361	9.0
23.1	7,121	11.7	26.8	3,073	10.1	30.5	451	8.9
23.2	7,004	11.7	26.9	2,972	10.1	30.6	540	8.9
23.3	6,887	11.7	27.0	2,871	10.1	30.7	629	8.8
23.4	6,770	11.6	27.1	2,770	10.0	30.8	717	8.8
23.5	6,554	11.6	27.2	2,670	10.0	30.9	805	− 8.8
23.6	6,538	− 11.5	27.3	2,570	− 10.0	31.0	− 893	
23.7	6,423		27.4	2,470				

* Compiled from Report of U. S. C. & G. Survey for 1881, App. 10, Table XI.

TABLE III.—COEFFICIENTS FOR CORRECTIONS FOR TEMPERATURE AND HUMIDITY.*

$t + t'$	*C*	Diff. for 1°.	$t + t'$	*C*	Diff. for 1°.	$t + t'$	*C*	Diff. for 1°.
0°	− .1024	10.9	60°	− .0380	10.7	120°	+ .0262	10.6
10	.0915	10.9	70.	.0273	10.7	130	.0368	10.4
20	.0606	10.8	80	.0166	10.8	140	.0472	10.3
30	.0698	10.6	90	− .0058	10.7	150	.0575	10.2
40	.0592	10.6	100	+ .0049	10.7	160	.0677	10.2
50	.0486	10.6	110	.0156	10.6	170	.0779	10.0
60	− .0380		120	+ .0262		180	+ .0879	

* *Compiled from Report of U. S. C. & G. Survey for 1881, App. 10, Tables I, IV.*

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POLAR PLANIMETER.

Equipment: Planimeter, sheet of drawing-paper about 15″ × 18″, two triangles, scale, drawing-compasses, *hard* pencil. All equipment except the planimeter should be furnished by the student.

Draw on the paper, with *extreme accuracy*, a circle 2″ in diameter, a circle 6″ in diameter, a 2″ square and a 4″ square. The 6″ circle should be nearly in the centre of the sheet, so that when the fixed point is placed inside the circle and the pointer traced around the circumference, the rolling wheel shall not run off the paper. Set the index of the arm of the planimeter at 295.5. With the pointer on some definite point of the perimeter of one of the figures read an initial reading (*not* necessarily zero) on the wheel and vernier. Move the pointer around the perimeter, turning to the right. When the starting-point is again reached, take the final reading. The difference in the readings divided by the known area in square inches gives the reading per square inch. Compute the quotient to three significant figures. Trace each figure similarly *twice*. The readings per square inch on all the figures should agree to a small fraction of one per cent.

Obtain the area of the "zero-circle" by taking the mean of two readings of the six-inch circle with the fixed point *inside* the circle, and then taking the *algebraic difference* of the mean readings (with the fixed point first inside, then outside), which difference, divided by the average reading per square inch, gives the area of the zero-circle in square inches. In taking the reading with the fixed point inside the circle, and the pointer turning as usual to the right, the index wheel will run *backward*, giving a *negative* quantity. Watch the index wheel carefully to note the whole number of revolutions. A simple way is to add, say, 30 to the reading, making it, say, 34.769; watch the wheel run backward through 0, and the readings are then in the twenties. Following it backward similarly until the pointer has again arrived at the starting-point of the perimeter and the reading, say 8.462, can be subtracted from 34.769, thus giving directly the total number of backward revolutions of the rolling wheel. This number is *negative*. The number obtained when the fixed point is outside the circle is *positive*. Therefore their algebraic difference is their numerical sum.

To compute the circumference of the rolling wheel. Use the formula $A = lnc$, in which $c =$ the required circumference, $n =$ the mean record of revolutions per square inch, $l = \left(\frac{295.5}{2}\right)^{mm} = 147.75mm = 5.817$ inches, $A = 1$ square inch.

NOTES.—The drawing, with all results suitably recorded on it. will constitute the report. In the field-book record simply the date, number of problem, etc.

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PANTOGRAPH.

Equipment: Pantograph, sheet of drawing-paper (detail paper will suffice) about 18″ × 24″, two triangles, scale, *hard* pencil. All equipment except the pantograph should be furnished by the student.

Use *extreme care* not to strain the arms or clamps while altering the ratios of the arm lengths. Record in each case by a sketch the arrangement of the fixed point, tracing point and pencil, and also the distances (measured to the nearest .01″) of these points from the pivots.

1. Set the instrument so that it will *magnify* 3.5 diameters and prove the work by making an enlarged copy of a 2″ square, and also of a 2″ circle.

2. Set the instrument so that it will *reduce* 2.5 diameters and prove the work by making a reduced copy of a 5″ square and also of a 5″ circle.

3. Set the instrument to retrace *at the same scale* any given figure and prove by applying it to some irregular figure sketched on the sheet.

NOTES.—The drawing, with all results suitably recorded, will constitute the report. In the field-book record simply date, number of problem, etc.

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RAILROAD SURVEYING—SIMPLE CURVES.

Equipment: Transit, 100-foot steel tape (or link-chain), two transit-poles, set of marking-pins.

Select a point from which may be run two lines making an angle of about 150° and each about 300 feet long. Let this point be the vertex of a curve with an assumed Δ (e.g., 28° 46′) and D (e.g., 5° 42′).

Compute the tangent distance. Set a pin at the vertex and measure along the direction of one tangent a distance equal to the tangent distance, which will give the P.C. Set up the transit at the P.C. and run in the curve to the P.T. Establish the direction of the forward tangent by a back-sight to the last transit station on the curve. If the vertex is visible from the P.T., sight directly to it and obtain the error (if any) in alignment; measure directly to it and compare the measurement with the computed tangent distance. If the P.T. and vertex are not intervisible, produce the forward tangent backward from the P.T. an amount equal to the tangent

Form of Notes.

Sta.	Alignment.	Vernier.	Tang. Defl.	Calc. Bearing.	Needle.
+02.3	P.T.	14° 23′	28° 46′	N 47° 10′ W	N 47° 15′ W
15	5° 42′ curve left for 28° 46′ Tan. dist. 257.88	14° 15′			
14		11° 24′			
13		8° 32′			
12		5° 42′			
11		5° 51′			
10	P.C.				
				N 18° 24′ W	N 18° 60′ W

distance, and observe the discrepancy between this point and the vertex.

The value of this exercise is increased when the whole curve is not visible from the P.C., thereby rendering necessary one or more transit-stations along the curve. In such cases, and especially when the degree of the curve is an odd quantity, the best plan is to compute all deflections as if the whole curve were visible from the P. C., checking the computation for the last substation by observing its equality to ½ Δ. With the transit at any intermediate station along the curve, instead of following the usual rule and having the verniers at zero when the telescope is tangent to the curve, point at the previous transit station with the verniers reading the *deflection for that station.* Plunge the telescope and point at any forward station with the same deflection as was computed for that station for the transit at P.C. In this way the original list of deflections can be followed and possible error avoided. For example (see below). with the transit at sta. 4, sight back to sta. 2 with a reading of 5° 42′. When the telescope is plunged and the verniers set at 11° 24′ the telescope will evidently be tangent to the curve. An additional deflection of 2° 51′ is necessary to point to sta. 5, and 14° 15′ is the computed deflection for sta. 5. The deflection for any station, visible or not, is the same as from the P.C.

(Read UP the page.)

Error of alignment, 0° 01⅛′

" " distance, 0′.2

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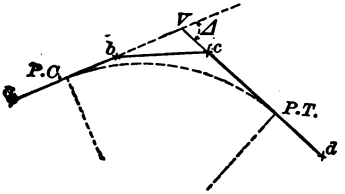
RAILROAD SURVEYING—SIMPLE CURVES, VERTEX INACCESSIBLE.

Equipment: Transit, 100-foot steel tape (or link-chain), two transit-poles, set of marking-pins.



Assume and locate the two tangents by setting two marking-pins for each, so that the central angle will be *about* 30°, and so that the two tangents would intersect within some building or other inaccessible place. Assume D at some value (e.g., 5° 26′) so that the curve will be about 500 to 600 feet long. Suppose *a*, *b*, *c*, and *d* are the marking-pins. Set up the transit at *b*; take a backsight to *a* and then point to *c*, thus measuring the angle *vbc*. Set up at *c* and similarly obtain the angle *bcv*. Measure *bc*, and then compute by trigonometry *vb* and *vc*. $vbc + bcv = \Delta$. From Δ and D the tangent distance may be found, and, knowing both this and *vb*, the P.C. may be located from *b*. Run in the curve to P.T. Check *alignment* by the direction of *cd*, and *distance* by measuring from *c* to P.T.

NOTES.—Similar to those for Prob. 25. Make a sketch (similar to that above) on the right-hand page, recording all the necessary computations.

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RAILROAD SURVEYING—TRANSITION CURVES.

(Searles' Railroad Spiral.)

Equipment: Transit, 100-foot steel tape (or link-chain), two transit-poles, set of marking-pins.

Select a suitable point as the vertex of a curve about 600 feet long, having a central angle of about 30°. Assume definite values for Δ and D and select a suitable spiral. Compute the tangent distance (T_s) for the curve and spiral. Lay off this tangent distance from the vertex; set up the transit at this point and run in the spiral. Set up the transit at the other end of the spiral and verify the setting of all the spiral points by back-sights. Run in the circular curve, locating its middle point (opposite the vertex), to the beginning of the terminal spiral. Set up the transit there and locate that spiral to the P.T. Finally set up at the P.T., verify the points of the terminal spiral, and check the location of the P.T. similarly to Prob. 25. Measure the distance from the middle point of the curve to the vertex and compare the measurement with the computed external distance (E_s).

NOTES.—Same form as for Prob. 25. The chord points of the spiral are recorded as substations with the proper deflection for each in the "vernier" column. The elements of the spiral chosen are recorded in the "alignment" column similarly to those for circular curves.

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RAILROAD SURVEYING—CROSS-SECTIONING.

Equipment: 50-foot tape, hand-level, 5-foot rod, level-rod (or light stadia-rod). ALSO, set of cross-section rods.*

1. **With hand-level.** Cross-section for 100 feet on each side of a line of stakes, already set by a previous survey. Obtain *distance* and *elevation* for every "break" in the slope, representing distance as the denominator of a fraction and elevation as the numerator. Elevation and distance are always referred to the centre.

2. **With cross-section rods.** Repeat the cross-sectioning, determining distance and elevation at every 10-foot point as well as at all breaks.

3. **Plot** one set of cross-sections on cross-section paper (10 lines to

FORM OF NOTES.

Sta.	Surf. Elev.	Grade Elev.	Center cut (+) or fill (–).	Grade.	
etc.	etc.	etc.	etc.		
+60	26.0	36.2	–10.2		
28	26.2	35.8	– 9.6		
27	38.4	25.2	+ 3.2		
26	42.7	34.6	+ 8.1		
25	44.6	34.0	+10.6	+0.6%	

---

* *The cross-section rods referred to* are a set of three rods, two of them 8 *feet long, divided into feet* and tenths, and used vertically. The third rod is 10 *feet long, divided also into feet* and tenths and provided with level-bubbles set

the inch, each way), using the scale of 10 feet to the inch, both for horizontal and vertical scales. The "elevations" (second column of NOTES), which are obtained from a previous levelling, are only used in the plotting. Using the heavy lines on the cross-section paper as even 10-foot elevations, plot first the elevation of "centre," and then, by addition or subtraction, the elevation of each point taken. A line drawn through these points will represent a cross-section of the surface. The intersection of this line by the 5 (10 or 20) foot lines of elevation gives graphically the distance of the 5 (10 or 20) foot contours from the centre. The "grade elevations" and "grade" may be assumed (for the sake of this problem) if they have not been definitely determined, and the "centre cut or fill" computed. Assume the width of road-bed as 20 feet in cut and 14 feet in fill. Assume slope ratio of 1½ horizontal to 1 vertical. Plot the outline of the cut (or embankment) on each cross-section.

4. Plot the centre line and contours on drawing-paper.

(Read UP the page.)

Left.	Right.
etc. $\frac{+14.6}{100}$ $\frac{+11.7}{60}$ $\frac{+9.8}{29.0}$ $\frac{+2.6}{13.6}$ $\frac{+2.1}{9.4}$ etc.	etc. $\frac{-3.3}{20.4}$ $\frac{-6.6}{45.0}$ $\frac{-9.7}{62.0}$ $\frac{-11.7}{100}$ etc.

in at each end. They are easily and cheaply made, and very rapid and accurate *work may be* done with them. On very rough ground, they are probably bette *than the hand-level* and tape. On smooth ground, the reverse is true. T *method of use* is obvious.

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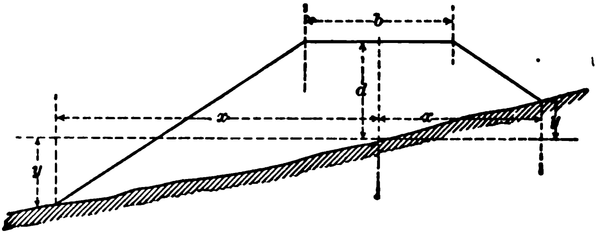
RAILROAD SURVEYING—SETTING SLOPE-STAKES.

Equipment: Level, level-rod (or light stadia-rod), 50-foot tape. Use stakes established by a previous survey, whose elevations are already determined. "Grade elevation" for each stake should also be known. Fill out all the columns on left-hand page of note-book (see below) before beginning field-work. Use the slope ratio $1\frac{1}{2}:1$; width of road-bed, 20 feet in cut, 14 feet in fill. Determine the distance and elevation of all "breaks" between the centre and the slope-stakes. Determine the location of the slope-stakes from the formula



$$x = \tfrac{1}{2}b + sd \pm sy.$$

At any station compute first $\frac{1}{2}b + sd$. Then, if on the lower side of a fill (as in figure), estimate by eye how much further than this a

FORM OF NOTES.

Sta.	Surf. Elev.	Grade Elev.	Centre cut (+) or fill (−)	Grade.	
21	*92.4*	*99.8*	— 7.4		

point would be so that the added distance would be 1½ times the difference of elevation of the point and the centre. Set up the rod at this total estimated distance. If the difference of elevation (y) satisfies the above formula, x being the horizontal distance from the centre, the point is the required position for the slope-stake. If the formula is not exactly satisfied, a little study will show whether the point should be nearer to or further from the centre and about how much. With practice, two trials will generally be sufficient to determine the point. The upper side of the fill is determined similarly, except that the correction (sy) is negative. For ground level transversely, sy vanishes.

An easy check on the numerical calculation is found by subtracting ½b from the computed distance and noting whether the difference is 1½ times the difference of elevation. For instance, take the result $\frac{3.0}{11.5}$. 11 5 − 7.0 = 4.5, which is 1½ times 3.0; again, 21.4 − 7.0 = 14.4, which is 1½ times 9.6.

(Read UP the page.)

Left.	Right. b = 14′ in fill, 20′ in cut s = 1½ : 1
$\frac{-3.0}{11.5}$ $\frac{-4.7}{8.0}$	$\frac{-8.6}{14.0}$ $\frac{-9.6}{21.4}$

PROBABLE ERROR—FORMULÆ AND USE.

[Nothing but a mere statement of working formulæ, with their use, is here given, which the student must take on trust if he has not studied the theory. The proof of these formulæ is taken up in Geodesy.]

1. When a number of separate observations of any kind have been made with equal care, their *arithmetical mean* is the *most probable value* of the quantity observed.

2. The *probable error* is such a quantity that it would be an even wager that it is greater or that it is less than the real error. For example, if the measurement of a base-line is given as 622.456 ± 0.096, it means that the arithmetical mean of several equally good measurements was 622.456; that the probable error (computed as below) is 0.096; that according to the mathematical laws of the theory of probability there is *as much chance* that the real error of 622.456 is *less* than 0.096 as that it is *more*.

3. Let n = the number of observations;

d = the difference between any one observation and the arithmetical mean (or the "weighted mean");

E_1 = the probable error of a single observation;

E_m = the probable error of the mean;

c = 0.6745, a constant determined by computation according to the theory;

Σ = a symbol signifying that a summation should be made of all the quantities represented by the following letter or letters.

Then

$$E_1 = c\sqrt{\frac{\Sigma d^2}{n-1}}$$

$$E_m = \frac{E_1}{\sqrt{n}} = c\sqrt{\frac{\Sigma d^2}{n(n-1)}}.$$

For example:

Angle.	d	d^2
66° 54′ 12″.5	+1.1	1.21
13 .5	+0.1	.01
11 .3	+2.3	5.29
16 .5	−2.9	8.41
12 .3	+1.3	1.69
15 .5	−1.9	3.61
Mean, 66° 54′ 13″.6		20.22

$$E_1 = .6745\sqrt{\frac{20.22}{6-1}} = \pm 1''.33.$$

$$E_m = .6745\sqrt{\frac{20.22}{6(6-1)}} = \pm 0''.54.$$

Angle = 66° 54′ 13″.6 ± 0″.54.

4. Weighted Observations. When observations have not been made with equal care or are not equally reliable they are "weighted" according to their reliability. If M represents any observation and w its weight, then the "weighted mean" $= \frac{\Sigma(wM)}{\Sigma(w)}$. Subtract each observation from the weighted mean to obtain d. Then, with the other symbols as before,

$$E_1 = c\sqrt{\frac{\Sigma(wd^2)}{n-1}} = \left\{\begin{array}{l}\text{the probable error of an observation of}\\ \text{weight unity;}\end{array}\right.$$

$$E_m = c\sqrt{\frac{\Sigma(wd^2)}{(\Sigma w)(n-1)}} = \text{the probable error of the weighted mean.}$$

For example:

Angle.	w	d	d^2	wd^2
116° 43′ 48″.81	5	+0.83	0.69	3.45
48 .76	4	+0.88	0.77	3.08
49 .53	5	+0.11	0.01	0.05
51 .56	3	−1.92	3.69	11.07
50 .38	2	−0.74	0.55	1.10
49 .84	5	−0.20	0.04	0.20
116° 43′ 49″.64				18.95

$$E_1 = 0.6745\sqrt{\frac{18.95}{6-1}} = 1''.32.$$

$$E_m = 0.6745\sqrt{\frac{18.95}{24(6-1)}} = 0''.27.$$

Angle = 116° 43′ 49″.64 ± 0″.27.

NOTES ON THE MANAGEMENT OF A STUDENTS' TOPOGRAPHICAL-HYDROGRAPHICAL SURVEY.

The following directions were originally written for the purpose of supplementing the general treatment of the subject, as given in a text-book on surveying, by specific instructions applied to the locality of the survey—a section of Fairmount Park, including a stretch of the Schuylkill River and a considerable area along the banks. They have been incorporated, with some revision, into this book because it is believed that they are sufficiently applicable to any similar survey to warrant their publication.

The triangulation methods are those for tertiary work, and the directions for "precise" base-line work, latitude and longitude, were inserted as exercises for the upper classmen, and not because they are essential to the proper conduct of the survey.

A. Reconnoissance; Location of Stations.

1. Two parties are necessary—one on each side of the river. Each party should be supplied with stakes, a heavy hatchet or axe, a pocket sextant (or prismatic compass), three transit-rods with flags tacked on, and a light crowbar.

2. In general four lines will run from each station—two to stations above and below on the same shore and two to stations on the opposite shore. The distance between stations along one shore should be, when possible, a little more than the width of the river so as to obtain nearly equilateral triangles.

3. The *forward* rodman on each side is controlled by the party on the *other* side. When the forward rodman reaches a suitable location that gives satisfactory angles (as near 60° as possible) to *both* parties he remains there until dismissed by *both* parties. The rear flagman on each side occupies the station last occupied by the main party.

4. The angles between stations should be measured with a pocket

sextant to the nearest degree. These should be plotted at night so that any necessary modifications can be promptly made, and then the stations can be properly numbered as a basis for future field orders.

5. All triangulation stations should be marked by stakes 2″ × 2″ × 18″, centred with tacks, driven to within 4″ of the ground, using a crowbar to make a hole if necessary. The stakes for the ends of the base-line should be longer; they should project at least 12″.

B. BASE-LINE. Precise Method; accuracy 1 : 1 000 000.

1. The site for the base-line should be nearly level, and on as uniform a surface as possible. The roadways on the river-banks generally afford the best sites.

2. A hub should be set every 300 feet, measuring from one end; also, a hub should be placed within 10 feet of the other end and at an even 10-foot distance from the last 300-foot hub placed. The hubs should be long enough to project 12″. At every 50 feet between the hubs 3-foot stakes should be driven so that the base-line would pass about ½″ from the face of the stake. Screw-eyes with 3″ hooks should be screwed into the faces of these stakes so that the bottom of each hook is on the grade-line between the tops of the adjacent hubs. The hooks should have the points bent upward, and the screws should be so set that the hooks will point outward and swing freely in both directions. Strips of zinc should be nailed to the tops of the hubs. While obtaining the grade-line between hubs, the differences of elevation of all adjacent hubs should be noted.

3. The tension-frames* should be used with a tension of 16 lbs. The rear end of the tape is placed exactly on the station mark, the forward end screwed up until the tension of 16 lbs. is obtained, every hook support critically examined to see that the tape rests with perfect freedom in the hooks, and then a fine mark is scratched in the zinc strip on the hub with a fine steel point. Then the tape is moved ahead. While measuring, two thermometers should be tied to the tape near the 100-ft. and 200-ft. points (marked 90 and 190 on the U. Pa. 300-foot tape), and the readings taken to the nearest

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* The tension-frames referred to were designed by the author. They provide for accurate adjustment of the tape over the hubs and an extremely accurate measurement of the tension—by a scale-beam and *not* by a spring. The frames are of tubing, with pointed ends, and are easily adjustable, tripod fashion, *over the hubs*.

tenth of a degree at every measurement. These thermometers should be carefully standardized and their exact errors known. The last sub-distance (less than 10 feet) is measured with a fine 10-foot steel tape, graduated to .01′ and estimated to .001′.

4. Measurements should not be made, unless for mere practice, while the sun is shining on the tape. A calm, cloudy day should be chosen, and even better results are obtained during a drizzling rain. In default of cloudy days, measurements should be taken immediately after sundown or before sunrise. For the best results the earth and atmosphere should have the same temperature. This probably occurs during the evening, but then lanterns must be used, endangering the accuracy.

C. BASE-LINE. Ordinary Method; probable accuracy, 1 : 50000.

1. Measure the base-line *three* times with an ordinary 100-foot steel tape, using fine brass plumb-bobs to plumb down the tape ends to the tops of stakes set every 100 feet. The mean temperature of the air should be noted with a pocket thermometer, and corrections made accordingly. Care should be used to have a uniform tension of as near 16 lbs. as possible. A spring-balance should be used for this.

2. The mean of these measurements may be used as the final value for the triangulation should anything prevent the "precise" measurement (B).

3. A check-base should be measured at or near the other end of the system of triangulation.

D. AZIMUTH.

1. Azimuth may be determined by a set of observations with a solar attachment. At least twenty observations should be obtained having an extreme range not greater than 8′ of arc (see Prob. 19). The observations should be taken at a triangulation station, and each observation consists in finding the angle between the meridian, as determined by that observation, and a line to another triangulation station.

2. Azimuth may also be determined by night observations on *Polaris (or 51 Cephei)* at elongation, or at *any* time provided that true *local time is accurately* known. (See Johnson's Surveying, § 381 a;

or Manual of Instructions, issued by the Commissioner of the General Land Office in 1890.)

3. A valuable check on the work may be obtained by taking azimuth at stations near each end of the triangulation, making corrections, if necessary, for the convergence of meridians.

4. True latitude (as close as the reading limit of the vertical arc of the transit) should be known. If it is not obtainable from an accurate map of the section of the country, it must be observed as provided below.

E. LATITUDE.

1. Observations for latitude may be made by observing the culmination of Polaris (or 51 Cephei) at the same time that the elongation of 51 Cephei (or Polaris) is observed for azimuth.

2. Latitude may also be obtained by transit (or sextant) observations on the sun at culmination, making allowance for the declination and refraction. (See Prob. 21, part 2.)

F. LONGITUDE.

1. Observations for longitude may be made by observing selected pairs of stars. (See Johnson's Surveying, §§ 382–392.) The true meridian should be accurately known for this work; a portable chronometer, with a known rate and a known error with reference to standard time, is also essential; the approximate longitude must also be known.

G. TRIANGULATION.

1. Sightings may be taken (1) at a transit-rod, held by a rodman, pointing as near the bottom of the rod as possible, noting by the vertical cross-hair the verticality of the rod, or (2) at a transit that may be occupying the station, pointing at the plumb-bob or at the vertical spindle underneath the plates. When it is evident that a station occupied by a transit party is being sighted at by another party, care must be taken to facilitate their work by not obstructing the line of view and always giving them a good mark to sight at. If a transit which is being sighted at is removed from the station, *it should be* promptly replaced by a rod.

2. *Angles should* be measured by the method outlined in Prob

13. Measure by repetition until six *consecutive* values are obtained having an extreme range not exceeding 2′ of arc. Compute the mean value.

3. All triangles must "close" with an error not exceeding 1′ of arc.

4. As fast as the angles of the triangles are *satisfactorily* measured, the sides should be computed from the measured base, the true azimuth of each triangulation line determined, and the whole triangulation plotted.

H. Levelling.

1. A line of levels should be run along each bank of the river, noting the elevation of each triangulation station and also of the water surface near each station.

2. A tide-gauge should be set up in a suitable place, the elevation of its zero determined, and readings to the nearest tenth of a foot taken as often as proves necessary, while the line of levels is run and while stadia topography is being taken.

I. Soundings.

1. Soundings to a depth of 18 to 20 feet may be made with a sounding-pole. A sounding-line with weight will be needed for a large part of the soundings above the Girard Ave. Bridge (max. depth about 36 feet).

2. Lines of soundings should be run nearly perpendicular to the course of the river and about 300 to 500 feet apart; the individual soundings about 50 feet apart.

3. The soundings should be located by two transits on shore pointing at each other and at the boat. As a check, a sextant in the boat points at the two transits. Each transit is set at 0° on the other transit, the lower plate always clamped, the upper plate free, the movement of the boat constantly followed with the telescope.

4. At the instant of taking a sounding, a signal will be given from the boat—a waving flag or a whistle. The shore parties will note *first* the *time* of the signal, and *second* the angle to the boat. *Since it will not increase* the accuracy of the plotting, the angles *need not be read closer* than the nearest 10′ or 15′, and so it will be *easier and better to take* the reading directly from the index, not

even reading the vernier. The transitmen should point at the sounding-pole as nearly as possible.

5. The shore parties and the boat party must be supplied with watches, previously compared to a close fraction of a minute. It will save trouble in identifying the shore and boat records if the soundings (and signals) are made always at an even minute of time, and never oftener than once per minute. An occasional signal of *two* whistles (marked as such in all the note-books) will also aid in identifying the soundings, especially when mistakes have been made.

6. The shore parties should be located at triangulation stations when they are suitable. Otherwise, points which are especially desirable for this purpose may be selected and their locations accurately determined from the triangulation stations.

7. The following form of notes should be used for the note-book in the sounding-boat:

Line.	Time.	Sextant Angle.	Angle.	Depth.	Bottom.
4—D_2	2:34 2:36	*E*—boat—*D*	62° 20′ 68° 40′	15.7 18.2	Sandy "

The right hand page should record the personnel of the party and any other desired information. D_2 means the second point on the shore below Sta. *D* toward (or away from) which a line of soundings has been run. The note-books of the shore parties should observe the following form:

Inst. at	Line.	Time.	Shore Angle.	Angle	
Δ *D*	4—D_2	2:34 2.36	boat—*D*—*E*	45° 10′ 42° 30′	

8. During the evening following the sounding work, the soundings in the sounding book should be numbered consecutively and the corresponding shore angles numbered with the same numbers, the correspondence being determined by the identity of time, verified by double signals, etc. These checks of double signals, etc., are often proved valuable.

J. TOPOGRAPHY.

1. Topography is taken by stadia lines, which must always begin and end on triangulation stations or other points equally well determined. Beginning at some station, the telescope should be pointed at some other known station, with the plates set at the known azimuth of that line.

2. Distance and vertical angle are read both for fore-sight and back-sight between stadia stations. Side shots are taken to all desired points within a radius of 500 to 600 feet.

3. One line should be run near enough to the shore line to take in all the configurations of the shore and also a belt of land from 200 to 600 feet wide—varying with the interference of trees and bluffs. The vertical angles to shore points should be noted with especial care, as it furnishes a valuable check on the levels.

4. The course of the other meander lines should depend largely on the topography. In general, they should follow roads, etc., so as to pass as near as possible to the important points that must be observed, and yet must pass near enough to the other lines that no feature of topography will be omitted.

5. If the transits contain fixed stadia wires, a careful test of them should be made at the beginning of the season's work, and if the interval does not correspond with the graduation of the rod, a reduction coefficient should be computed and applied to each reading. If the wires are adjustable, tests should be made frequently, even daily, considering how simple it is. Measure off a base-line 600 feet long on level ground near headquarters. Each morning, before beginning stadia work, set up the transit at a distance "$f + c$" behind one end of the base-line, and read the rod held at the other end. An error of more than 1 : 600 should be immediately adjusted, after consultation with an instructor. If the first "set-up" of the transit is to be at a triangulation station, it may be more convenient to begin there at once and point to another station whose distance is known, and take the reading, making due allowance for the "$f + c$."

6. Further instructions on this subject, together with a form of notes, may be found in the instructions for a preliminary survey *(railroad work) on pp. 58–61.*

K. COMPUTATIONS; PLOTTING.

1. The closure of triangles within the required limits must be determined as soon as possible, and the angles remeasured if necessary.

2. The azimuth and length of all triangulation lines must be computed as soon as sufficient data are obtained.

3. To check topographic work, the co-ordinates of all triangulation stations from some chosen meridian and parallel should be computed before topographic work commences.

4. The latitudes and departures of the lines between stations of the topographic work should be computed and checked with the previously computed co-ordinates of the triangulation stations to which the stadia lines run.

5. For such lines as are run on this survey—probably less than ½ mile between such stations as are used to check on—the errors should be within 10 feet horizontal distance and 1 foot in vertical height.* Greater errors than these call for a thorough revision of notes to discover a possible error, and perhaps a resurvey.

6. The triangulation should be plotted to a scale of 200 feet to the inch as soon as the computations are complete.

7. All soundings should be plotted during the progress of the survey, or at least sufficient of them to show the general accuracy and reliability of the notes.

8. All topographical notes should be plotted during the survey (at least in pencil)—plotting first the transit stations until the line checks with sufficient accuracy on some known station and then plotting the side shots, locating all drives, walks, buildings, rail roads, bridges, etc.

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* These errors are very large considering what accuracy is possible with the stadia, but it is doubtful if much closer work than this can be expected of inexperienced students.

NOTES ON THE MANAGEMENT OF A STUDENTS' RAILROAD SURVEY.

It should be remembered that students' surveys are made primarily to teach the students. Therefore, although it is desirable to simulate regular professional practice as far as possible, matters of speed and economy should sometimes be subordinated, so that the students may receive the maximum benefit.

RECONNOISSANCE.

A reconnoissance survey is a very hasty examination of a wide belt of country to determine which routes will best justify further and closer instrumental examination. In practice they vary from a trip made on horseback, depending principally on "judgment" and "an eye for country," to an instrumental survey differing but little in character from a preliminary survey. The prime object is to determine with the greatest rapidity and least expense the salient features which will condemn one proposed route and commend another, or which will narrow down the choice to two or three lines that are so evenly balanced that a closer survey of each is necessary to decide between them. In places where the topographical maps of the U. S. Geological Survey (or other similar maps) are available, no reconnoissance (at least so far as the *physical* characteristics of the country are concerned) is necessary; a general route may be chosen from the map and the preliminary survey at once made. In all civilized countries some sort of a map can generally be found, although coarse and inaccurate. The method here outlined presupposes the availability of a map which will give the relative *horizontal* positions of prominent features of the country, such as cross-roads, streams, etc., with such a degree of accuracy as county maps, as ordinarily made, are found to have. In the absence of such a map a better method of obtaining distances should be adopted.

The instrumental equipment consists of a mercurial barometer, an aneroid, a pocket thermometer, a pocket compass, and a hand-level. The mercurial barometer should be maintained at headquarters and read every half-hour during the periods when the aneroid is being read in the field. The altitude above sea-level of each location for headquarters should be obtained if possible by a comparisome with some known altitude, points on another railroad, a geodetic survey station, etc. If nothing of the kind is available, extraordinary care should be used in obtaining the difference of elevation of the different locations for headquarters.

Distances may be estimated, paced (aided perhaps by a pedometer), or measured when convenient with an odometer. Azimuth may be measured with a pocket compass, and altitudes with the aneroid.

The survey line should start from some known place on the map (generally some town), and should pass through all points which will probably be determining points on the road. In the course of a few miles it will probably pass within range of some easily identified point—a road crossing, bridge, hamlet, etc. "Side shots" should be taken to all such points and to any points whose position might have some bearing on the proper location of the road. Sketches should be made on the right-hand pages of the note-book on a scale of 400 feet to the inch, sketching not only the topographical features, but also the *form* of the contours. During the evenings following each day's work in reconnoissance the aneroid observations should be reduced (by comparison with the plotted mercurial readings for the day) and entered in the note-book. (See Prob. 22.) Then the line should be plotted on the map of the country that has been obtained for a basis, modifying the location of the points by making the plotting of well-defined points agree with the (probably) more correct map used as a basis and drawing in contours as well as the data will permit, utilizing the aneroid elevations. From this rough map may be decided what line (or lines) will justify a further and closer examination by means of the "preliminary survey."

FORM OF NOTES. RECONNOISSANCE SURVEY.

Observer at	Bearing.	Distance.	Aneroid.	Reduced Elevation.	Sighting at
	S 70° W	500′			Hill about 90′ high.
	N 5° E	600′			Road corner.
			29″.922 64°	....	Creek under bridge.
	N 5° E	50′			bridge.
	N 25° W	950′			△ 23
△ 22			29″.860 66°	....	

PRELIMINARY SURVEY.

The party should consist of a transitman, recorder, two rodmen with stadia-rods, stakeman, leveller, and level-rod man, and a topographer.*

Azimuth at the first station should be determined as accurately as possible by a needle-reading, setting vernier A at zero, with the needle pointing south. A splendid check on azimuth may be obtained by using a solar attachment at the beginning and end of the survey, and also at intermediate points at intervals of a few miles, making due allowance for the "convergence of meridians." The extra time required for these check observations is insignificant. Carry the traverse throughout the survey by the same method as in Prob. 18.

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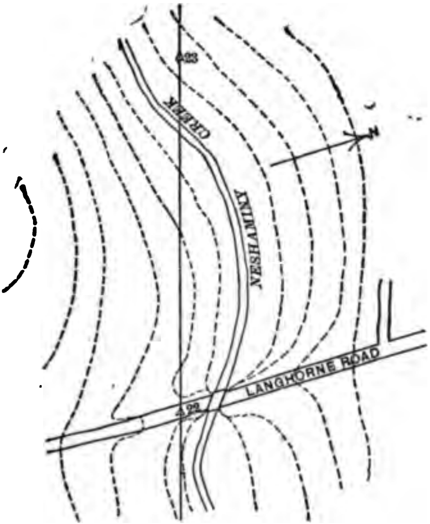
*See note on page 61.

Read up the page.



Particular care should be taken in observing the distances b tween stations. Not only should the distance be read both durir fore-sight and back-sight observations, but if the sighting is not vei clear for any reason repeated sightings should be taken as suggeste in Prob. 17. If the back-sight does not agree with the previous for sight, the error should be located until there is no doubt of the tr distance.

Take "side shots" to all fence corners, road bends, river bend buildings, salient points of ground surface, and any other poin necessary to obtain a topographical map with 5-foot contou *throughout the* belt of country traversed by the survey. The w *of country* necessary to be surveyed varies with the topograph should include a considerable width each side of any probable location of the road in that belt. Even when topographical features determine the location to within a few feet, the topography should be taken for 200 feet on *each side*. From this minimum width it should be increased up to 1000 feet each side as the uncertainty of possible location increases. Much time and work can be saved by limiting the observations that are taken only for contours to points that are the salient points—in the bed of a stream or dry gully, the comb of a ridge, the edge of a bluff, etc.

The elevations of all stations should be determined with a wye-level to the nearest tenth of a foot. Benches and turning points should be read to the nearest hundredth. A bench-mark should be established every half-mile.

The topographer should plot his work on section-paper at a scale of 200 feet to the inch, plotting first each station point and then the position of the more prominent stadia points, using azimuths and unreduced stadia readings as aids in plotting distances; using the level notes of elevation of stations to determine where the 5-foot contours cross the traverse line, observing (with hand-level) the course of each contour throughout the area included by the survey. A sketch-board, about 18″ square, mounted on a very light tripod, is very useful for this work. The board should be oriented at each station. A ruler may be used as an alidade, as in plane-table work, and a very reliable and valuable sketch may be rapidly made. Of course this sketch is not expected to take the place of the office map, but it will render the office plotting very easy and certain.

FORM OF NOTES. PRELIMINARY SURVEY.

	Vern. A.	Needle.	Wire Int.	Vert. Angle.	Red. Dist.	Diff. Elev.	Elevation.
F. S.—Δ 27	62° 47	S 62° 45 W	5.68	+ 0° 52′	564	+ 8.5	172.76
	....	——	....	....	....	....	....
	etc	——	etc.	etc.	'etc.	etc.	etc.
	322° 14′	——	8.47	+ 3° 21′	844	+ 49.6	2.14
B. S.—Δ 23	265° 18′	N 85° 15′ E	4.22	− 0° 16′	423	− 2.0	——
Inst. at Δ 23							164.28

During the evening following each day's work in the preliminary survey the "reduced distances," "differences of elevation," and "elevations" are to be computed from the notes and entered in the note-book. Then the work is to be plotted on detail paper to a scale of 200 feet to the inch. The "paper location" should then be studied and drawn on the preliminary map. From the intersection of this line with the contours an approximate profile may be drawn which will be valuable in giving an idea of the amount of cut and fill, and will be especially valuable in comparison with one or more profiles similarly obtained from other proposed locations.

LOCATION SURVEY.

The location party may consist of a transitman, a recorder, two tapemen, two rodmen, a stakeman, two cross-section men, a leveller, and a level-rod man. When short of men the transitman may keep notes and one rodman may be dispensed with.*

Stakes should be located at every station (at least) on tangents, every 50 feet on curves up to 6°, every 25 feet on sharper curves; also at all abrupt changes in the profile (both to increase accuracy in chaining and to locate points for the leveller). Stakes should be marked on the *rear* side, with the number of the station and its plus-distance, if any. Transit hubs should be driven flush with the ground, the exact centre marked with a tack, and should have a witness-stake set three feet to the left, with the distance and P.C. or P.T., etc., marked on it, the marking facing the hub.

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* To render these student surveys as free from objection (and expenses for damages) as possible, cutting and clearing is avoided wherever possible. With a superabundance of men, branches can be held back and cutting often avoided. Therefore no reference has been made to axemen, either in this or in the preliminary survey. The force of men is ample to supply an axeman for all cutting that is indispensable. The necessity for clearing is allowed considerable weight in selecting the locality for these surveys.

(Read up the page.)

(level, 172.73.)

to fence cor. near barn.

The leveller should observe the elevation of all stations and substations, and should even interpolate points with a tape and observe their elevations when it will add to the accuracy of the profile. The B.M.'s of the preliminary survey should be used and checked as long as they are within range of the location line. New B.M.'s may be interpolated when necessary. The rod should be read to the nearest tenth on stations and to the nearest hundredth on turning-points and bench-marks. The profile of each day's work should be promptly plotted at night on profile-paper, Plate A, using horizontal scale 200 feet to the inch, vertical scale 20 feet to the inch.

FORM OF NOTES. LOCATION SURVEY.

Sta.	Alignment.	Vernier.	Tang. Defl.	Calc.Bearing.	Needle.
△ + 47	P.T.	6° 10′	12° 20′	N 1° 24′ E	N 1° 30′ E
52	4° curve right for 12° 20′ Tang. dist. = 154.80	5° 13½′			
51		3° 13½′			
50		1° 13′			
△ + 39	P.C.				
49					
48					
△ + 50	P.T.	7° 20′	14° 40′	N 10° 56′ W	N 11° 0′ W
47	2° 40′ curve left for 14° 40′ Tang. dist. = 276.54.	6° 40′			
△ + 50		6° 00′			
46		5° 20′			
45		4° 00′			
△ + 50		3° 20′			
44		2° 40′			
43		1° 20′			
△ *42*	*P.C.*			N 3° 44′ E	N 3° 40′ E

The transitman should " tie on " to the stakes of the preliminary survey, whenever possible, not only as a check on the work, but also to observe and correct any departure from the line as laid out on account of inaccuracies in the paper location.

The cross-section men should work by the methods and with the form of notes elaborated in Prob. 28. The width of the cross-sections may be varied according to the amount of cut or fill that would evidently be required. They should be made about 50 feet wider than the probable width of top of cut or base of fill. They should be plotted at night on cross-section paper.

The location map should be plotted at night to a scale of 200 feet to the inch, drawing in 5-foot contours from the cross-section notes and borrowing other topographical details not observed during the location survey from the preliminary survey map.

The form of notes for the location survey should be as on pp. 62, 63:

(Read UP the page.)

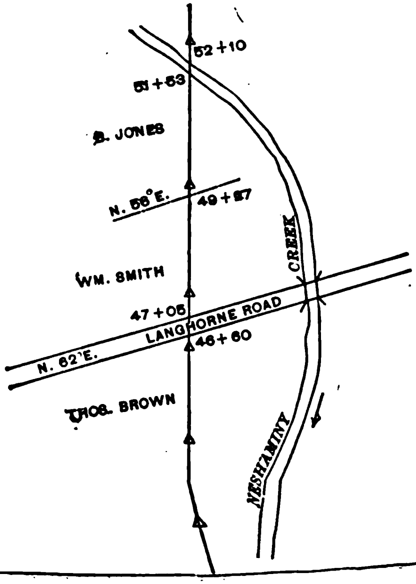

TABLE IV.—SQUARES OF NUMBERS.

	**0	**1	**2	**3	**4	**5	**6	**7	**8	**9
10	10 000	10 201	10 404	10 609	10 816	11 025	11 236	11 449	11 664	11 881
11	12 100	12 321	12 544	12 769	12 996	13 225	13 456	13 689	13 924	14 161
12	14 400	14 641	14 884	15 129	15 376	15 625	15 876	16 129	16 384	16 641
13	16 900	17 161	17 424	17 689	17 956	18 225	18 496	18 769	19 044	19 321
14	19 600	19 881	20 164	20 449	20 736	21 025	21 316	21 609	21 904	22 201
15	22 500	22 801	23 104	23 409	23 716	24 025	24 336	24 649	24 964	25 281
16	25 600	25 921	26 244	26 569	26 896	27 225	27 556	27 889	28 224	28 561
17	28 900	29 241	29 584	29 929	30 276	30 625	30 976	31 329	31 684	32 041
18	32 400	32 761	33 124	33 489	33 856	34 225	34 596	34 969	35 344	35 721
19	36 100	36 481	36 864	37 249	37 636	38 025	38 416	38 809	39 204	39 601
20	40 000	40 401	40 804	41 209	41 616	42 025	42 436	42 849	43 264	43 681
21	44 100	44 521	44 944	45 369	45 796	46 225	46 656	47 089	47 524	47 961
22	48 400	48 841	49 284	49 729	50 176	50 625	51 076	51 529	51 984	52 441
23	52 900	53 361	53 824	54 289	54 756	55 225	55 696	56 169	56 664	57 121
24	57 600	58 081	58 564	59 049	59 536	60 025	60 516	61 009	61 504	62 001
25	62 500	63 001	63 504	64 009	64 516	65 025	65 536	66 049	66 564	67 081
26	67 600	68 121	68 644	69 169	69 696	70 225	70 756	71 289	71 824	72 361
27	72 900	73 441	73 984	74 529	75 076	75 625	76 176	76 729	77 284	77 841
28	78 400	78 961	79 524	80 089	80 656	81 225	81 796	82 369	82 944	83 521
29	84 100	84 681	85 264	85 849	86 436	87 025	87 616	88 209	88 804	89 401
30	90 000	90 601	91 204	91 809	92 416	93 025	93 636	94 249	94 864	95 481
31	96 100	96 721	97 344	97 969	98 596	99 225	99 856	100 489	101 124	101 761
32	102 400	103 041	103 684	104 329	104 976	105 625	106 276	106 929	107 584	108 241
33	108 900	109 561	110 224	110 889	111 556	112 225	112 896	113 569	114 244	114 921
34	115 600	116 281	116 964	117 649	118 336	119 025	119 716	120 409	121 104	121 801
35	122 500	123 201	123 904	124 609	125 316	126 025	126 736	127 449	128 164	128 881
36	129 600	130 321	131 044	131 769	132 496	133 225	133 956	134 689	135 424	136 161
37	136 900	137 641	138 384	139 129	139 876	140 625	141 376	142 129	142 884	143 641
38	144 400	145 161	145 924	146 689	147 456	148 225	148 996	149 769	150 544	151 321
39	152 100	152 881	153 664	154 449	155 236	156 025	156 816	157 609	158 404	159 201
40	160 000	160 801	161 604	162 409	163 216	164 025	164 836	165 649	166 464	167 281
41	168 100	168 921	169 744	170 569	171 396	172 225	173 056	173 889	174 724	175 561
42	176 400	177 241	178 084	178 929	179 776	180 625	181 476	182 329	183 184	184 041
43	184 900	185 761	186 624	187 489	188 356	189 225	190 096	190 969	191 844	192 721
44	193 600	194 481	195 364	196 249	197 136	198 025	198 916	199 809	200 704	201 601
45	202 500	203 401	204 304	205 209	206 116	207 025	207 936	208 849	209 764	210 681
46	211 600	212 521	213 444	214 369	215 296	216 225	217 156	218 089	219 024	219 961
47	220 900	221 841	222 784	223 729	224 676	225 625	226 576	227 529	228 484	229 441
48	230 400	231 361	232 324	233 289	234 256	235 225	236 196	237 169	238 144	239 121
49	240 100	241 081	242 064	243 049	244 036	245 025	246 016	247 009	248 004	249 001
50	250 000	251 001	252 004	253 009	254 016	255 025	256 036	257 049	258 064	259 081
51	260 100	261 121	262 144	263 169	264 196	265 225	266 256	267 289	268 324	269 361
52	270 400	271 441	272 484	273 529	274 576	275 625	276 676	277 729	278 784	279 841
53	280 900	281 961	283 024	284 089	285 156	286 225	287 296	288 369	289 444	290 521
54	291 600	292 681	293 764	294 849	295 936	297 025	298 116	299 209	300 304	301 401
55	302 500	303 601	304 704	305 809	306 916	308 025	309 136	310 249	311 364	312 481
56	313 600	314 721	315 844	316 969	318 096	319 225	320 356	321 489	322 624	323 761
57	324 900	326 041	327 184	328 329	329 476	330 625	331 776	332 929	334 084	335 241
58	336 400	337 561	338 724	339 889	341 056	342 225	343 396	344 569	345 744	346 921
59	348 100	349 281	350 464	351 649	352 836	354 025	355 216	356 409	357 604	358 801

TABLE IV.—SQUARES OF NUMBERS.

	..0	..1	..2	..3	..4	..5	..6	..7	..8	..9
10	10 000	10 201	10 404	10 609	10 816	11 025	11 236	11 449	11 664	11 881
11	12 100	12 321	12 544	12 769	12 996	13 225	13 456	13 689	13 924	14 161
12	14 400	14 641	14 884	15 129	15 376	15 625	15 876	16 129	16 384	16 641
13	16 900	17 161	17 424	17 689	17 956	18 225	18 496	18 769	19 044	19 321
14	19 600	19 881	20 164	20 449	20 736	21 025	21 316	21 609	21 904	22 201
15	22 500	22 801	23 104	23 409	23 716	24 025	24 336	24 649	24 964	25 281
16	25 600	25 921	26 244	26 569	26 896	27 225	27 556	27 889	28 224	28 561
17	28 900	29 241	29 584	29 929	30 276	30 625	30 976	31 329	31 684	32 041
18	32 400	32 761	33 124	33 489	33 856	34 225	34 596	34 969	35 344	35 721
19	36 100	36 481	36 864	37 249	37 636	38 025	38 416	38 809	39 204	39 601
20	40 000	40 401	40 804	41 209	41 616	42 025	42 436	42 849	43 264	43 681
21	44 100	44 521	44 944	45 369	45 796	46 225	46 656	47 089	47 524	47 961
22	48 400	48 841	49 284	49 729	50 176	50 625	51 076	51 529	51 984	52 441
23	52 900	53 361	53 824	54 289	54 756	55 225	55 696	56 169	56 644	57 121
24	57 600	58 081	58 564	59 049	59 536	60 025	60 516	61 009	61 504	62 001
25	62 500	63 001	63 504	64 009	64 516	65 025	65 536	66 049	66 564	67 081
26	67 600	68 121	68 644	69 169	69 696	70 225	70 756	71 289	71 824	72 361
27	72 900	73 441	73 984	74 529	75 076	75 625	76 176	76 729	77 284	77 841
28	78 400	78 961	79 524	80 089	80 656	81 225	81 796	82 369	82 944	83 521
29	84 100	84 681	85 264	85 849	86 436	87 025	87 616	88 209	88 804	89 401
30	90 000	90 601	91 204	91 809	92 416	93 025	93 636	94 249	94 864	95 481
31	96 100	96 721	97 344	97 969	98 596	99 225	99 856	100 489	101 124	101 761
32	102 400	103 041	103 684	104 329	104 976	105 625	106 276	106 929	107 584	108 241
33	108 900	109 561	110 224	110 889	111 556	112 225	112 896	113 569	114 244	114 921
34	115 600	116 281	116 964	117 649	118 336	119 025	119 716	120 409	121 104	121 801
35	122 500	123 201	123 904	124 609	125 316	126 025	126 736	127 449	128 164	128 881
36	129 600	130 321	131 044	131 769	132 496	133 225	133 956	134 689	135 424	136 161
37	136 900	137 641	138 384	139 129	139 876	140 625	141 376	142 129	142 884	143 641
38	144 400	145 161	145 924	146 689	147 456	148 225	148 996	149 769	150 544	151 321
39	152 100	152 881	153 664	154 449	155 236	156 025	156 816	157 609	158 404	159 201
40	160 000	160 801	161 604	162 409	163 216	164 025	164 836	165 649	166 464	167 281
41	168 100	168 921	169 744	170 569	171 396	172 225	173 056	173 889	174 724	175 561
42	176 400	177 241	178 084	178 929	179 776	180 625	181 476	182 329	183 184	184 041
43	184 900	185 761	186 624	187 489	188 356	189 225	190 096	190 969	191 844	192 721
44	193 600	194 481	195 364	196 249	197 136	198 025	198 916	199 809	200 704	201 601
45	202 500	203 401	204 304	205 209	206 116	207 025	207 936	208 849	209 764	210 681
46	211 600	212 521	213 444	214 369	215 296	216 225	217 156	218 089	219 024	219 961
47	220 900	221 841	222 784	223 729	224 676	225 625	226 576	227 529	228 484	229 441
48	230 400	231 361	232 324	233 289	234 256	235 225	236 196	237 169	238 144	239 121
49	240 100	241 081	242 064	243 049	244 036	245 025	246 016	247 009	248 004	249 001
50	250 000	251 001	252 004	253 009	254 016	255 025	256 036	257 049	258 064	259 081
51	260 100	261 121	262 144	263 169	264 196	265 225	266 256	267 289	268 324	269 361
52	270 400	271 441	272 484	273 529	274 576	275 625	276 676	277 729	278 784	279 841
53	280 900	281 961	283 024	284 089	285 156	286 225	287 296	288 369	289 444	290 521
54	291 600	292 681	293 764	294 849	295 936	297 025	298 116	299 209	300 304	301 401
55	302 500	303 601	304 704	305 809	306 916	308 025	309 136	310 249	311 364	312 481
56	313 600	314 721	315 844	316 969	318 096	319 225	320 356	321 489	322 624	323 761
[illegible]	[illegible] 900	326 041	327 184	328 329	329 476	330 625	331 776	332 929	334 084	335 241
[illegible]	[illegible]	337 561	338 724	339 889	341 056	342 225	343 396	344 569	345 744	346 921
[illegible]	[illegible]	349 281	350 464	351 649	352 836	354 025	355 216	356 409	357 604	358 801

TABLE IV.—SQUARES OF NUMBERS.—(*Concluded.*)

	**0	**1	**2	**3	**4	**5	**6	**7	**8	**9
60	360 000	361 201	362 404	363 609	364 816	366 025	367 236	368 449	369 664	370 881
61	372 100	373 321	374 544	375 769	376 996	378 225	379 456	380 689	381 924	383 161
62	384 400	385 641	386 884	388 129	389 376	390 625	391 876	393 129	394 384	395 641
63	396 900	398 161	399 424	400 689	401 956	403 225	404 496	405 769	407 044	408 321
64	409 600	410 881	412 164	413 449	414 736	416 025	417 316	418 609	419 904	421 201
65	422 500	423 801	425 104	426 409	427 716	429 025	430 336	431 649	432 964	434 281
66	435 600	436 921	438 244	439 569	440 896	442 225	443 556	444 889	446 224	447 561
67	448 900	450 241	451 584	452 929	454 276	455 625	456 976	458 329	459 684	461 041
68	462 400	463 761	465 124	466 489	467 856	469 225	470 596	471 969	473 344	474 721
69	476 100	477 481	478 864	480 249	481 636	483 025	484 416	485 809	487 204	488 601
70	490 000	491 401	492 804	494 209	495 616	497 025	498 436	499 849	501 264	502 681
71	504 100	505 521	506 944	508 369	509 796	511 225	512 656	514 089	515 524	516 961
72	518 400	519 841	521 284	522 729	524 176	525 625	527 076	528 529	529 984	531 441
73	532 900	534 361	535 824	537 289	538 756	540 225	541 696	543 169	544 644	546 121
74	547 600	549 081	550 564	552 049	553 536	555 025	556 516	558 009	559 504	561 001
75	562 500	564 001	565 504	567 009	568 516	570 025	571 536	573 049	574 564	576 081
76	577 600	579 121	580 644	582 169	583 696	585 225	586 756	588 289	589 824	591 361
77	592 900	594 441	595 984	597 529	599 076	600 625	602 176	603 729	605 284	606 841
78	608 400	609 961	611 524	613 089	614 656	616 225	617 796	619 369	620 944	622 521
79	624 100	625 681	627 264	628 849	630 436	632 025	633 616	635 209	636 804	638 401
80	640 000	641 601	643 204	644 809	646 416	648 025	649 636	651 249	652 864	654 481
81	656 100	657 721	659 344	660 969	662 596	664 225	665 856	667 489	669 124	670 761
82	672 400	674 041	675 684	677 329	678 976	680 625	682 276	683 929	685 584	687 241
83	688 900	690 561	692 224	693 889	695 556	697 225	698 896	700 569	702 244	703 921
84	705 600	707 281	708 964	710 649	712 336	714 025	715 716	717 409	719 104	720 801
85	722 500	724 201	725 904	727 609	729 316	731 025	732 736	734 449	736 164	737 881
86	739 600	741 321	743 044	744 769	746 496	748 225	749 956	751 689	753 424	755 161
87	756 900	758 641	760 384	762 129	763 876	765 625	767 376	769 129	770 884	772 641
88	774 400	776 161	777 924	779 689	781 456	783 225	784 996	786 769	788 544	790 321
89	792 100	793 881	795 664	797 449	799 236	801 025	802 816	804 609	806 404	808 201
90	810 000	811 801	813 604	815 409	817 216	819 025	820 836	822 649	824 464	826 281
91	828 100	829 921	831 744	833 569	835 396	837 225	839 056	840 889	842 724	844 561
92	846 400	848 241	850 084	851 929	853 776	855 625	857 476	859 329	861 184	863 041
93	864 900	866 761	868 624	870 489	872 356	874 225	876 096	877 969	879 844	881 721
94	883 600	885 481	887 364	889 249	891 136	893 025	894 916	896 809	898 704	900 601
95	902 500	904 401	906 304	908 209	910 116	912 025	913 936	915 849	917 764	919 681
96	921 600	923 521	925 444	927 369	929 296	931 225	933 156	935 089	937 024	938 961
97	940 900	942 841	944 784	946 729	948 676	950 625	952 576	954 529	956 484	958 441
98	960 400	962 361	964 324	966 289	968 256	970 225	972 196	974 169	976 144	978 121
99	980 100	982 081	984 064	986 049	988 036	990 025	992 016	994 009	996 004	998 001

TABLE V.—LOGARITHMS OF NUMBERS.

	0	1	2	3	4	5	6	7	8	9	Diff.
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	42
11	414	453	492	531	569	607	645	682	719	755	38
12	792	828	864	899	934	969	1004	1038	1072	1106	35
13	1139	1173	1206	1239	1271	1303	335	367	399	430	32
14	461	492	523	553	584	614	644	673	703	732	30
15	761	790	818	847	875	903	931	959	987	2014	28
16	2041	2068	2095	2122	2148	2175	201	2227	2253	279	26
17	304	330	355	380	405	430	455	480	504	529	25
18	553	577	601	625	648	672	695	718	742	765	24
19	788	810	833	856	878	900	923	945	967	989	22
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	21
21	222	243	263	284	304	324	345	365	385	404	20
22	424	444	464	483	502	522	541	560	579	598	19
23	617	636	655	674	692	711	729	747	766	784	
24	802	820	838	856	874	892	909	927	945	962	18
25	979	997	4014	4031	4048	4065	4082	4099	4116	4133	17
26	4150	4166	183	200	216	232	249	265	281	298	16
27	314	330	346	362	378	393	409	425	440	456	
28	472	487	502	518	533	548	564	579	594	609	15
29	624	639	654	669	683	698	713	728	742	757	
30	771	786	800	814	829	843	857	871	886	900	14
31	914	928	942	955	969	983	997	5011	5024	5038	
32	5051	5065	5079	5092	5105	5119	5182	145	159	172	13
33	185	198	211	224	237	250	263	276	289	302	
34	315	328	340	353	366	378	391	403	416	428	
35	441	453	465	478	490	502	515	527	539	551	12
36	563	575	587	599	611	623	635	647	658	670	
37	682	694	705	717	729	740	752	763	775	786	
38	798	809	821	832	843	855	866	877	888	899	
39	911	922	933	944	955	966	977	988	999	6010	11
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	117	
41	128	138	149	159	170	180	191	201	212	222	
42	232	243	253	263	274	284	294	304	314	325	
43	335	345	355	365	375	385	395	405	415	425	10
44	435	444	454	464	474	484	493	503	513	522	
45	532	542	551	561	571	580	590	599	609	618	
46	628	637	646	656	665	675	684	693	702	712	
47	721	730	739	749	758	767	776	785	794	803	
48	812	821	830	839	848	857	866	875	884	893	9
49	902	911	920	928	937	946	955	964	972	981	
50	990	998	7007	7016	7024	7033	7042	7050	7059	7067	
51	7076	7084	093	101	110	118	126	135	143	152	
52	160	168	177	185	193	202	210	218	226	235	
53	243	251	259	267	275	284	292	300	308	316	8
54	324	332	340	348	356	364	372	380	388	396	
	0	1	2	3	4	5	6	7	8	9	

TABLE V.—LOGARITHMS OF NUMBERS.—(*Concluded.*)

	0	1	2	3	4	5	6	7	8	9	Diff.
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	8
56	482	490	497	505	513	520	528	536	543	551	
57	559	566	574	582	589	597	604	612	619	627	
58	634	642	649	657	664	672	679	686	694	701	
59	709	716	723	731	738	745	752	760	767	774	
60	782	789	796	803	810	818	825	832	839	846	
61	853	860	868	875	882	889	896	903	910	917	7
62	924	931	938	945	952	959	966	973	980	987	
63	993	8000	8007	8014	8021	8028	8035	8041	8048	8055	
64	8062	069	075	082	089	096	102	109	116	122	
65	129	136	142	149	156	162	169	176	182	189	
66	195	202	209	215	222	228	235	241	248	254	
67	261	267	274	280	287	293	299	306	312	319	
68	325	331	338	344	351	357	363	370	376	382	
69	388	395	401	407	414	420	426	432	439	445	
70	451	457	463	470	476	482	488	494	500	506	
71	513	519	525	531	537	543	549	555	561	567	6
72	573	579	585	591	597	603	609	615	621	627	
73	633	639	645	651	657	663	669	675	681	686	
74	692	698	704	710	716	722	727	733	739	745	
75	751	756	762	768	774	779	785	791	797	802	
76	808	814	820	825	831	837	842	848	854	859	
77	865	871	876	882	887	893	899	904	910	915	
78	921	927	932	938	943	949	954	960	965	971	
79	976	982	987	993	998	9004	9009	9015	9020	9025	
80	9031	9036	9042	9047	9053	058	063	069	074	079	
81	085	090	096	101	106	112	117	122	128	133	
82	138	143	149	154	159	165	170	175	180	186	
83	191	196	201	206	212	217	222	227	232	238	
84	243	248	253	258	263	269	274	279	284	289	
85	294	299	304	309	315	320	325	330	335	340	
86	345	350	355	360	365	370	375	380	385	390	5
87	395	400	405	410	415	420	425	430	435	440	
88	445	450	455	460	465	469	474	479	484	489	
89	494	499	504	509	513	518	523	528	533	538	
90	542	547	552	557	562	566	571	576	581	586	
91	590	595	600	605	609	614	619	624	628	633	
92	638	643	647	652	657	661	666	671	675	680	
93	685	689	694	699	703	708	713	717	722	727	
94	731	736	741	745	750	754	759	763	768	773	
95	777	782	786	791	795	800	805	809	814	818	
96	823	827	832	836	841	845	850	854	859	863	
97	868	872	877	881	886	890	894	899	903	908	
98	912	917	921	926	930	934	939	943	948	952	
99	956	961	965	969	974	978	983	987	991	996	4
	0	1	2	3	4	5	6	7	8	9	

TABLE VI.—LOGARITHMIC TRIGONOMETRIC FUNCTIONS.

	0°		1°			2°			
′	Sin Tan	Cot	Sin	Tan	Cot	Sin	Tan	Cot	′
0	− ∞	+ ∞	8.2419	8.2419	11.7581	8.5428	8.5431	11.4569	60
1	6.4637	13.5363	2490	2491	7509	5464	5467	4533	59
2	7648	2352	2561	2562	7438	5500	5503	4497	58
3	9408	0592	2630	2631	7369	5535	5538	4462	57
4	7.0658	12.9342	2699	2700	7300	5571	5573	4427	56
5	1627	8373	2766	2767	7233	5605	5608	4392	55
6	2419	7581	2832	2833	7167	5640	5643	4357	54
7	3088	6912	2898	2899	7101	5674	5677	4323	53
8	3668	6332	2962	2963	7037	5708	5711	4289	52
9	4180	5820	3025	3026	6974	5742	5745	4255	51
10	4637	5363	3088	3089	6911	5776	5779	4221	50
11	7.5051	12.4949	8.3150	8.3150	11.6850	8.5809	8.5812	11.4188	49
12	5429	4571	3210	3211	6789	5842	5845	4155	48
13	5777	4223	3270	3271	6729	5875	5878	4122	47
14	6099	3901	3329	3330	6670	5907	5911	4089	46
15	6398	3602	3388	3389	6611	5939	5943	4057	45
16	6678	3322	3445	3446	6554	5972	5975	4025	44
17	6942	3058	3502	3503	6497	6003	6007	3993	43
18	7190	2810	3558	3559	6441	6035	6038	3962	42
19	7425	2575	3613	3614	6386	6066	6070	3930	41
20	7648	2352	3668	3669	6331	6097	6101	3899	40
21	7.7859	12.2140	8.3722	8.3723	11.6277	8.6128	8.6132	11.3868	39
22	8061	1938	3775	3776	6224	6159	6163	3837	38
23	8255	1745	3828	3829	6171	6189	6193	3807	37
24	8439	1561	3880	3881	6119	6220	6223	3777	36
25	8617	1383	3931	3932	6068	6250	6254	3746	35
26	8787	1213	3982	3983	6017	6279	6283	3717	34
27	8951	1049	4032	4033	5967	6309	6313	3687	33
28	9109	0891	4082	4083	5917	6339	6343	3657	32
29	9261	0739	4131	4132	5868	6368	6372	3628	31
30	9408	0591	4179	4181	5819	6397	6401	3599	30
31	7.9551	12.0449	8.4227	8.4229	11.5771	8.6426	8.6430	11.3570	29
32	9689	0311	4275	4276	5724	6454	6459	3541	28
33	9822	0177	4322	4323	5677	6483	6487	3513	27
34	9952	0048	4368	4370	5630	6511	6515	3485	26
35	8.0078	11.9922	4414	4416	5584	6539	6544	3456	25
36	0200	9800	4459	4461	5539	6567	6571	3429	24
37	0319	9681	4504	4506	5494	6595	6599	3401	23
38	0435	9565	4549	4551	5449	6622	6627	3373	22
39	0548	9452	4593	4595	5405	6650	6654	3346	21
40	0658	9342	4637	4638	5362	6677	6682	3318	20
′	Cos Cot	Tan	Cos	Cot	Tan	Cos	Cot	Tan	′
	89°		88°			87°			

TABLE VI.—LOGARITHMIC TRIGONOMETRIC FUNCTIONS.—(*Continued.*)

	0°		1°			2°			
′	Sin Tan	Cot	Sin	Tan	Cot	Sin	Tan	Cot	′
41	8.0765	11.9235	8.4680	8.4682	11.5318	8.6704	8.6709	11.3291	19
42	0870	9130	4723	4725	5275	6731	6736	3264	18
43	0972	9028	4765	4767	5233	6758	6762	3238	17
44	1072	8928	4807	4809	5191	6784	6789	3211	16
45	1169	8830	4848	4850	5150	6810	6815	3185	15
46	1265	8735	4890	4892	5108	6837	6842	3158	14
47	1358	8641	4930	4932	5068	6863	6868	3132	13
48	1450	8550	4971	4973	5027	6889	6894	3106	12
49	1539	8460	5011	5013	4987	6914	6920	3080	11
50	1627	8373	5050	5053	4947	6940	6945	3055	10
51	8.1713	11.8287	8.5090	8.5092	11.4908	8.6965	8.6971	11.3029	9
52	1797	8204	5129	5131	4869	6991	6996	3004	8
53	1880	8120	5167	5170	4830	7016	7021	2979	7
54	1961	8039	5206	5208	4792	7041	7046	2954	6
55	2041	7959	5243	5246	4754	7066	7071	2929	5
56	2119	7880	5281	5283	4717	7090	7096	2904	4
57	2196	7804	5318	5321	4679	7115	7121	2879	3
58	2271	7729	5355	5358	4642	7140	7145	2855	2
59	2346	7654	5392	5394	4606	7164	7170	2830	1
60	8.2419	11.7581	8.5428	8.5431	11.4569	8.7188	8.7194	11.2806	0
′	Cos Cot	Tan	Cos	Cot	Tan	Cos	Cot	Tan	′
	89°		88°			87°			

COSINES. 0° to 3°.

′	0°	1°	2°	′
0	10.0000	9.9999	9.9997	60
10	0000	9999	9997	50
20	0000	9999	9996	40
30	0000	9999	9996	30
40	0000	9998	9995	20
50	0000	9998	9995	10
60	9.9999	9.9997	9.9994	0
′	89°	88°	87°	′

TABLE VI.—LOGARITHMIC TRIGONOMETRIC FUNCTIONS.—(*Continued.*

° ′	Sin	Diff. 1′	Cos	Tan	Diff. 1′	Cot	° ′
3°	8.7188	23.5	9.9994	8·7194	23.5	11.2806	87°
10	7423	22.3	9993	7429	22.3	2571	50
20	7645	21.2	9993	7652	21.2	2348	40
30	7857	20.2	9992	7865	20.3	2135	30
40	8059	19.3	9991	8067	19.4	1933	20
50	8251	18.5	9990	8261	18.5	1739	10
4°	8.8436	17.7	9.9989	8.8446	17.8	11.1554	86°
10	8613	17.0	9989	8624	17.1	1376	50
20	8783	16.3	9988	8795	16.5	1205	40
30	8946	15.8	9987	8960	15.8	1040	30
40	9104	15.2	9986	9118	15.3	0882	20
50	9256	14.7	9985	9272	14.8	0728	10
5°	8.9403	14.2	9.9983	8.9420	14.3	11.0580	85°
10	9545	13.7	9982	9563	13.8	0437	50
20	9682	13.4	9981	9701	13.4	0299	40
30	9816	12.9	9980	9836	13.0	0164	30
40	9945	12.5	9979	9966	12.7	0034	20
50	9.0070	12.2	9977	9.0093	12.3	10.9907	10
6°	9.0192	11.9	9.9976	9.0216	12.0	10.9784	84°
10	0311	11.5	9975	0336	11.7	9664	50
20	0426	11.2	9973	0453	11.4	9547	40
30	0539	10.9	9972	0567	11.1	9433	30
40	0648	10.7	9971	0678	10.8	9322	20
50	0755	10.4	9969	0786	10.5	9214	10
7°	9.0859	10.2	9.9968	9.0891	10.4	10.9109	83°
10	0961	9.9	9966	0995	10.1	9005	50
20	1060	9.7	9964	1096	9.8	8904	40
30	1157	9.5	9963	1194	9.7	8806	30
40	1252	9.3	9961	1291	9.4	8709	20
50	1345	9.1	9959	1385	9.3	8615	10
8°	9.1436	8.9	9.9958	9.1478	9.1	10.8522	82°
10	1525	8.7	9956	1569	8.9	8431	50
20	1612	8.5	9954	1658	8.7	8342	40
30	1697	8.4	9952	1745	8.6	8255	30
40	1781	8.2	9950	1831	8.4	8169	20
50	1863	8.0	9948	1915	8.2	8085	10
9°	9.1943	7.9	9.9946	9.1997	8.1	10.8003	81°
10	2022	7.8	9944	2078	8.0	7922	50
20	2100	7.6	9942	2158	7.8	7842	40
30	2176	7.5	9940	2236	7.7	7764	30
40	2251	7.3	9938	2313	7.6	7687	20
50	2324	7.2	9936	2389	7.4	7611	10
10°	9.2397		9.9934	9.2463		10.7537	80°
° ′	Cos	Diff. 1′	Sin	Cot	Diff. 1′	Tan	° ′

TABLE VI.—LOGARITHMIC TRIGONOMETRIC FUNCTIONS.—(*Continued.*)

° ′	Sin	Diff. 1′	Cos	Tan	Diff. 1′	Cot	° ′
10°	9.2397	7.1	9.9934	9.2463	7.3	10.7537	80°
10	2468	7.0	9931	2536	7.3	7464	50
20	2538	6.9	9929	2609	7.1	7391	40
30	2606	6.8	9927	2680	7.0	7320	30
40	2674	6.7	9924	2750	6.9	7250	20
50	2740	6.6	9922	2819	6.8	7181	10
11°	9.2806	6.4	9.9919	9.2887	6.7	10.7113	79°
10	2870	6.4	9917	2953	6.6	7047	50
20	2934	6.3	9914	3020	6.5	6980	40
30	2997	6.2	9912	3085	6.4	6915	30
40	3058	6.1	9909	3149	6.3	6851	20
50	3119	6.0	9907	3212	6.2	6788	10
12°	9.3179	5.9	9.9904	9.3275	6.1	10.6725	78°
10	3238	5.8	9901	3336	6.1	6664	50
20	3296	5.7	9899	3397	6.0	6603	40
30	3353	5.7	9896	3458	5.9	6542	30
40	3410	5.6	9893	3517	5.9	6483	20
50	3466	5.5	9890	3576	5.8	6424	10
13°	9.3521	5.4	9.9887	9.3634	5.7	10.6366	77°
10	3575	5.4	9884	3691	5.7	6309	50
20	3629	5.3	9881	3748	5.6	6252	40
30	3682	5.2	9878	3804	5.5	6196	30
40	3734	5.2	9875	3859	5.5	6141	20
50	3786	5.1	9872	3914	5.4	6086	10
14°	9.3837	5.0	9.9869	9.3968	5.3	10.6032	76°
10	3887	5.0	9866	4021	5.3	5979	50
20	3937	4.9	9863	4074	5.2	5926	40
30	3986	4.9	9859	4127	5.2	5873	30
40	4035	4.8	9856	4178	5.1	5822	20
50	4083	4.7	9853	4230	5.1	5770	10
15°	9.4130	4.7	9.9849	9.4281	5.0	10.5719	75°
10	4177	4.6	9846	4331	5.0	5669	50
20	4223	4.6	9843	4381	4.9	5619	40
30	4269	4.5	9839	4430	4.9	5570	30
40	4314	4.5	9836	4479	4.8	5521	20
50	4359	4.4	9832	4527	4.8	5473	10
16°	9.4403	4.4	9.9828	9.4575	4.7	10.5425	74°
10	4447	4.4	9825	4622	4.7	5378	50
20	4491	4.3	9821	4669	4.7	5331	40
30	4533	4.2	9817	4716	4.6	5284	30
40	4576	4.2	9814	4762	4.6	5238	20
50	4618	4.1	9810	4808	4.5	5192	10
17°	9.4659		9.9806	9.4853		10.5147	73°
° ′	Cos	Diff. 1′	Sin	Cot	Diff. 1′	Tan	° ′

TABLE VI.—LOGARITHMIC TRIGONOMETRIC FUNCTIONS.—(*Continued.*)

° ′	Sin	Diff. 1′	Cos	Tan	Diff. 1′	Cot	° ′
17°	9.4659	4.1	9.9806	9.4853	4.5	10.5147	73°
10	4700	4.1	9802	4898	4.5	5102	50
20	4741	4.0	9798	4943	4.4	5057	40
30	4781	4.0	9794	4987	4.4	5013	30
40	4821	4.0	9790	5031	4.4	4969	20
50	4861	3.9	9786	5075	4.3	4925	10
18°	9.4900	3.9	9.9782	9.5118	4.3	10.4882	72°
10	4939	3.8	9778	5161	4.2	4839	50
20	4977	3.8	9774	5203	4.2	4797	40
30	5015	3.8	9770	5245	4.2	4755	30
40	5052	3.7	9765	5287	4.2	4713	20
50	5090	3.7	9761	5329	4.1	4671	10
19°	9.5126	3.7	9.9757	9.5370	4.1	10.4630	71°
10	5163	3.6	9752	5411	4.0	4589	50
20	5199	3.6	9748	5451	4.0	4549	40
30	5235	3.6	9743	5491	4.0	4509	30
40	5270	3.5	9739	5531	4.0	4469	20
50	5306	3.5	9734	5571	4.0	4429	10
20°	9.5341	3.5	9.9730	9.5611	3.9	10.4389	70°
10	5375	3.4	9725	5650	3.9	4350	50
20	5409	3.4	9721	5689	3.9	4311	40
30	5443	3.4	9716	5727	3.8	4273	30
40	5477	3.3	9711	5766	3.8	4234	20
50	5510	3.3	9706	5804	3.8	4196	10
21°	9.5543	3.3	9.9702	9.5842	3.8	10.4158	69°
10	5576	3.3	9697	5879	3.8	4121	50
20	5609	3.2	9692	5917	3.7	4083	40
30	5641	3.2	9687	5954	3.7	4046	30
40	5673	3.2	9682	5991	3.7	4009	20
50	5704	3.1	9677	6028	3.6	3972	10
22°	9.5736	3.1	9.9672	9.6064	3.6	10.3936	68°
10	5767	3.1	9667	6100	3.6	3900	50
20	5798	3.1	9661	6136	3.6	3864	40
30	5828	3.0	9656	6172	3.6	3828	30
40	5859	3.0	9651	6208	3.5	3792	20
50	5889	3.0	9646	6243	3.5	3757	10
23°	9.5919	3.0	9.9640	9.6279	3.5	10.3721	67°
10	5948	2.9	9635	6314	3.5	3686	50
20	5978	2.9	9629	6348	3.5	3652	40
30	6007	2.9	9624	6383	3.4	3617	30
40	6036	2.9	9618	6417	3.4	3583	20
50	6065	2.8	9613	6452	3.4	3548	10
24°	9.6093		9.9607	9.6486		10.3514	66°
° ′	Cos	Diff. 1′	Sin	Cot	Diff. 1′	Tan	° ′

TABLE VI.—LOGARITHMIC TRIGONOMETRIC FUNCTIONS.—(*Continue*

° ′	Sin	Diff. 1′	Cos	Tan	Diff. 1′	Cot	° ′
24°	9.6093	2.8	9.9607	9.6486	3.4	10.3514	66°
10	6121	2.8	9602	6520	3.4	3480	50
20	6149	2.8	9596	6553	3.4	3447	40
30	6177	2.8	9590	6587	3.3	3413	30
40	6205	2.7	9584	6620	3.3	3380	20
50	6232	2.7	9579	6654	3.3	3346	10
25°	9.6259	2.7	9.9573	9.6687	3.3	10.3313	65°
10	6286	2.7	9567	6720	3.3	3280	50
20	6313	2.7	9561	6752	3.3	3248	40
30	6340	2.6	9555	6785	3.2	3215	30
40	6366	2.6	9549	6817	3.2	3183	20
50	6392	2.6	9543	6850	3.2	3150	10
26°	9.6418	2.6	9.9537	9.6882	3.2	10.3118	64°
10	6444	2.6	9530	6914	3.2	3086	50
20	6470	2.5	9524	6946	3.2	3054	40
30	6495	2.5	9518	6977	3.2	3023	30
40	6521	2.5	9512	7009	3.1	2991	20
50	6546	2.5	9505	7040	3.1	2960	10
27°	9.6570	2.5	9.9499	9.7072	3.1	10.2928	63°
10	6595	2.5	9492	7103	3.1	2897	50
20	6620	2.4	9486	7134	3.1	2866	40
30	6644	2.4	9479	7165	3.1	2835	30
40	6668	2.4	9473	7196	3.1	2804	20
50	6692	2.4	9466	7226	3.1	2774	10
28°	9.6716	2.4	9.9459	9.7257	3.0	10.2743	62°
10	6740	2.4	9453	7287	3.0	2713	50
20	6763	2.3	9446	7317	3.0	2683	40
30	6787	2.3	9439	7348	3.0	2652	30
40	6810	2.3	9432	7378	3.0	2622	20
50	6833	2.3	9425	7408	3.0	2592	10
29°	9.6856	2.3	9.9418	9.7438	3.0	10.2562	61°
10	6878	2.3	9411	7467	3.0	2533	50
20	6901	2.2	9404	7497	3.0	2503	40
30	6923	2.2	9397	7526	2.9	2474	30
40	6946	2.2	9390	7556	2.9	2444	20
50	6968	2.2	9383	7585	2.9	2415	10
30°	9.6990	2.2	9.9375	9.7614	2.9	10.2386	60°
10	7012	2.2	9368	7644	2.9	2356	50
20	7033	2.1	9361	7673	2.9	2327	40
30	7055	2.1	9353	7701	2.9	2299	30
40	7076	2.1	9346	7730	2.9	2270	20
50	7097	2.1	9338	7759	2.9	2241	10
31°	9.7118		9.9331	9.7788		10.2212	59°
° ′	Cos	Diff. 1′	Sin	Cot	Diff. 1′	Tan	° ′

TABLE VI.—LOGARITHMIC TRIGONOMETRIC FUNCTIONS.—(*Continued.*)

° ′	Sin	Diff. 1′	Cos	Diff. 1′	Tan	Diff. 1′	Cot	° ′
31°	9.7118	2.1	9.9331	0.8	9.7788	2.9	10.2212	**59°**
10	7139	2.1	9323	0.8	7816	2.8	2184	50
20	7160	2.1	9315	0.8	7845	2.8	2155	40
30	7181	2.1	9308	0.8	7873	2.8	2127	30
40	7201	2.0	9300	0.8	7902	2.8	2098	20
50	7222	2.0	9292	0.8	7930	2.8	2070	10
32°	9.7242	2.0	9.9284	0.8	9.7958	2.8	10.2042	**58°**
10	7262	2.0	9276	0.8	7986	2.8	2014	50
20	7282	2.0	9268	0.8	8014	2.8	1986	40
30	7302	2.0	9260	0.8	8042	2.8	1958	30
40	7322	2.0	9252	0.8	8070	2.8	1930	20
50	7342	2.0	9244	0.8	8097	2.8	1903	10
33°	9.7361	1.9	9.9236	0.8	9.8125	2.8	10.1875	**57°**
10	7380	1.9	9228	0.8	8153	2.8	1847	50
20	7400	1.9	9219	0.8	8180	2.7	1820	40
30	7419	1.9	9211	0.8	8208	2.7	1792	30
40	7438	1.9	9203	0.8	8235	2.7	1765	20
50	7457	1.9	9194	0.8	8263	2.7	1737	10
34°	9.7476	1.9	9.9186	0.9	9.8290	2.7	10.1710	**56°**
10	7494	1.9	9177	0.9	8317	2.7	1683	50
20	7513	1.8	9169	0.9	8344	2.7	1656	40
30	7531	1.8	9160	0.9	8371	2.7	1629	30
40	7550	1.8	9151	0.9	8398	2.7	1602	20
50	7568	1.8	9142	0.9	8425	2.7	1575	10
35°	9.7586	1.8	9.9134	0.9	9.8452	2.7	10.1548	**55°**
10	7604	1.8	9125	0.9	8479	2.7	1521	50
20	7622	1.8	9116	0.9	8506	2.7	1494	40
30	7640	1.8	9107	0.9	8533	2.7	1467	30
40	7657	1.8	9098	0.9	8559	2.7	1441	20
50	7675	1.7	9089	0.9	8586	2.7	1414	10
36°	9.7692	1.7	9.9080	0.9	9.8613	2.7	10.1387	**54°**
10	7710	1.7	9070	0.9	8639	2.6	1361	50
20	7727	1.7	9061	0.9	8666	2.6	1334	40
30	7744	1.7	9052	0.9	8692	2.6	1308	30
40	7761	1.7	9042	0.9	8718	2.6	1282	20
50	7778	1.7	9033	0.9	8745	2.6	1255	10
37°	9.7795	1.7	9.9023	1.0	9.8771	2.6	10.1229	**53°**
10	7811	1.7	9014	1.0	8797	2.6	1203	50
20	7828	1.7	9004	1.0	8824	2.6	1176	40
30	7844	1.6	8995	1.0	8850	2.6	1150	30
40	7861	1.6	8985	1.0	8876	2.6	1124	20
50	7877	1.6	8975	1.0	8902	2.6	1098	10
38°	9.7893		9.8965		9.8928		10.1072	**52°**
	Cos	Diff. 1′	Sin	Diff. 1′	Cot	Diff. 1′	Tan	° ′

TABLE VI.—LOGARITHMIC TRIGONOMETRIC FUNCTIONS.—(*Concluded*

° ′	Sin	Diff. 1′	Cos	Diff. 1′	Tan	Diff. 1′	Cot	° ′
38°	9.7893	1.6	9.8965	1.0	9.8928	2.6	10.1072	52°
10	7910	1.6	8955	1.0	8954	2.6	1046	50
20	7926	1.6	8945	1.0	8980	2.6	1020	40
30	7941	1.6	8935	1.0	9006	2.6	0994	30
40	7957	1.6	8925	1.0	9032	2.6	0968	20
50	7973	1.6	8915	1.0	9058	2.6	0942	10
39°	9.7989	1.6	9.8905	1.0	9.9084	2.6	10.0916	51°
10	8004	1.5	8895	1.0	9110	2.6	0890	50
20	8020	1.5	8884	1.0	9135	2.6	0865	40
30	8035	1.5	8874	1.0	9161	2.6	0839	30
40	8050	1.5	8864	1.1	9187	2.6	0813	20
50	8066	1.5	8853	1.1	9212	2.6	0788	10
40°	9.8081	1.5	9.8843	1.1	9.9238	2.6	10.0762	50°
10	8096	1.5	8832	1.1	9264	2.6	0736	50
20	8111	1.5	8821	1.1	9289	2.6	0711	40
30	8125	1.5	8810	1.1	9315	2.6	0685	30
40	8140	1.5	8800	1.1	9341	2.6	0659	20
50	8155	1.5	8789	1.1	9366	2.6	0634	10
41°	9.8169	1.4	9.8778	1.1	9.9392	2.6	10.0608	49°
10	8184	1.4	8767	1.1	9417	2.5	0583	50
20	8198	1.4	8756	1.1	9443	2.5	0557	40
30	8213	1.4	8745	1.1	9468	2.5	0532	30
40	8227	1.4	8733	1.1	9494	2.5	0506	20
50	8241	1.4	8722	1.1	9519	2.5	0481	10
42°	9.8255	1.4	9.8711	1.1	9.9544	2.5	10.0456	48°
10	8269	1.4	8699	1.1	9570	2.5	0430	50
20	8283	1.4	8688	1.2	9595	2.5	0405	40
30	8297	1.4	8676	1.2	9621	2.5	0379	30
40	8311	1.4	8665	1.2	9646	2.5	0354	20
50	8324	1.4	8653	1.2	9671	2.5	0329	10
43°	9.8338	1.4	9.8641	1.2	9.9697	2.5	10.0303	47°
10	8351	1.3	8629	1.2	9722	2.5	0278	50
20	8365	1.3	8618	1.2	9747	2.5	0253	40
30	8378	1.3	8606	1.2	9772	2.5	0228	30
40	8391	1.3	8594	1.2	9798	2.5	0202	20
50	8405	1.3	8582	1.2	9823	2.5	0177	10
44°	9.8418	1.3	9.8569	1.2	9.9848	2.5	10.0152	46°
10	8431	1.3	8557	1.2	9874	2.5	0126	50
20	8444	1.3	8545	1.2	9899	2.5	0101	40
30	8457	1.3	8532	1.2	9924	2.5	0076	30
40	8469	1.3	8520	1.3	9949	2.5	0051	20
50	8482	1.3	8507	1.3	9975	2.5	0025	10
45°	9.8495		9.8495		10.0000		10.0000	45°
° ′	Cos	Diff. 1′	Sin	Diff. 1′	Cot	Diff. 1′	Tan	° ′

TABLE VI.—LOGA

° ′	Sin
31°	9.7118
10	7139
20	7160
30	7181
40	7201
50	7222
32°	9.7242
10	7262
20	7282
30	7302
40	7322
50	7342
33°	9.7361
10	7380
20	7400
30	7419
40	7438
50	7457
34°	9.7476
10	7494
20	7513
30	7531
40	7550
50	7568
35°	9.7586
10	7604
20	7622
30	7640
40	7657
50	7675
36°	9.7692
10	7710
20	[illegible]
30	
40	
50	
37	
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